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Probing naked singularities in non-supersymmetric string vacua

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Abstract: We present a detailed analysis of non-supersymmetric spacetime varying string vacua which can lead to an exponential hierarchy between the electroweak and the gravitational scales. In particular, we identify a limit in which these vacua can be interpreted as supersymmetric vacua of F-theory. Furthermore, we study the properties of these solutions as seen by D7-brane probes and establish a non-supersymmetric analogue of the enhançon mechanism.

Keywords: Extra Large Dimensions, p-branes, Superstring Vacua.

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Contents

1. Introduction

In a recent paper [1] we considered $p+1$-dimensional cosmic defects (and in particular $p = D - 3$) embedded in a D-dimensional spacetime.[1] It was explicitly shown how these cosmic branes correspond to non-supersymmetric string vacua which naturally can lead to an exponential hierarchy between the D and $D-2$ dimensional scales. These types of models have been considered before in the literature [15]–[18]. The exponential hierarchy between the electroweak and gravitational scales arises from non-trivial warp factors in the metric and is naturally generated by the string coupling of $O(1)$. (The rôle of warp factors in string theory and their relationship to cosmic brane models have been studied in [19]–[22].)

As pointed out in [1], our solutions resemble the stringy cosmic strings of [23, 24]. The general framework is that of a higher-dimensional string theory compactified on a Calabi-Yau (complex) n-fold, M_n, some moduli of which are allowed to vary over part of the non-compact space. In the uncompactified type-IIB theory, the rôle of space-dependent moduli is played by the dilaton-axion system, very much like in Vafa's description of F-theory [25]. The cosmic defect (brane) appears as a singularity of the induced spacetime metric, with its characteristics governed by the energy momentum tensor of the moduli. In addition, there is a naked singularity, located at a finite proper distance from the core of the brane, which is the source of the exponential hierarchy.

Our solutions do not saturate the BPS bound in general, supersymmetry being broken in the presence of the naked singularity, and can be thought of as a particular example of warped Kaluza-Klein compactifications in string theory. However, one of the main points of this paper is the claim that there exists a one-parameter family of spacetime varying vacua in IIB string theory, which naturally leads to an exponential hierarchy and which at one point in the parameter space can be identified with particular vacua of F-theory.

The singular properties of the metric of our solution, responsible for the exponential hierarchy, can be resolved in string theory via a non-supersymmetric effect analogous to the enhançon mechanism. This mechanism is based on a repulsive nature of the naked singularity as seen by quantum probes of the background. This last fact should be contrasted to the usual resolution of classically repulsive singularities. Our results offer an exciting possibility that supersymmetry breaking is related to the emergence of an exponentially large hierarchy.

The paper is organized as follows: in section 2, we recall the construction of spacetime varying string vacua and generalize the ansätze of refs. [23, 24, 25] for the metric; we find non-trivial non-supersymmetric solutions in which the moduli are non-holomorphic functions over the noncompact space. We concentrate on a

[1] The possibility that our $3+1$-dimensional world can be viewed as a cosmic defect (brane) in a higher-dimensional theory has been studied by many authors, for example [2]–[14].

single modulus scenario, in which the modulus is given by the dilaton-axion system of $D = 10$ type-IIB string theory, even though we can in general treat the dynamics of moduli arising from generic non-supersymmetric compactifications of type-I, type-IIA, or heterotic string theory.

In particular, we present a general solution for the single modulus scenario in type-IIB string theory in which the axion-dilaton system is only a function of the angular coordinate of the transverse space. For simplicity we take the longitudinal part of the metric for our solution to be conformally flat, even though a more general class of solutions can be found. We give a detailed account of the spacetime properties of this particular kind of solution; the source-free case corresponds to a neutral black 7-brane (analogous to the Schwarzschild black hole in $D = 4$), while the charged brane, which represents a non-trivial axion/dilaton configuration, is obtained by adding 7-branes to the neutral background.

Our solution possesses a classical naked singularity which is responsible for the existence of an exponential hierarchy. We conduct both classical and quantum analysis of the effective dynamics of a test probe in the background of such a naked singularity. While the classical analysis indicates that the naked singularity is attractive, the quantum treatment of the same problem reveals that the singularity is reflective, and that the whole spacetime looks like a box with impenetrable walls placed at the positions of the classically naked singularities.

Finally we study a supersymmetric limit of our solution and its interpretation as a particular vacuum of F-theory. We are guided by the similarity between the transverse parts of the corresponding metrics for the supersymmetric and non-supersymmetric solutions. We also point out the crucial role played by the $\mathrm{SL}(2, \mathbb{Z})$ symmetry of IIB string theory in establishing this limit.

In order to understand the supersymmetric limit of our solution as well as the nature of the naked singularities from the string theory point of view, in section 3, we probe the geometry of our solution with D7-branes while in section 4, we consider a non-supersymmetric effect analogous to the enhançon mechanism. While physics is in general dependent on the nature of the probe, we find that the results of the probe analysis are in accord with the results of section 2: a quantum probe of our background must remain in a scattered state although the effective potential exerted by the classically naked singularity is attractive. We repeat the probe analysis after wrapping our ten-dimensional background on a K3, which is potentially relevant from a purely phenomenological view-point, and find that the naked singularites while present in supergravity are resolved in string theory in such a way that the exponential hierarchy is preserved.

2. The cosmic brane solution

In this section we review the solution of [1] and discuss in detail its most general form.

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2.1 The general problem

Let us consider compactifications of string theory in which the "internal" space (a Calabi-Yau n-fold M_n) varies over the "observable" spacetime. The parameters of the "internal" space then become spacetime variable moduli fields ϕ^α. The effective action describing the coupling of moduli to gravity of the observable spacetime can be derived by dimensionally reducing the higher dimensional Einstein-Hilbert action [23, 24]. In this procedure one retains the dependence of the Ricci scalar in the gravitational action only on the moduli ϕ^α. Thus, the relevant part of the low-energy effective D-dimensional action of the moduli of the Calabi-Yau n-fold, M_n, coupled to gravity reads[2]

$$S_{\rm eff} = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\sqrt{-g}\left(R - G_{\alpha\bar\beta}g^{\mu\nu}\partial_\mu\phi^\alpha\partial_\nu\phi^{\bar\beta} + \cdots\right), \tag{2.1}$$

where $\mu,\nu = 0,\ldots,D-1$, and $2\kappa^2 = 16\pi G_N^D$, where G_N^D is the D-dimensional Newton constant. We neglect higher derivative terms and set the other fields in the theory to zero as in [23].

We will restrict the moduli to depend on x_i, $i = D-2, D-1$, so that $\partial_a\phi = 0$, $a = 0,\ldots,D-3$. The equations of motion are

$$g^{ij}\left(\nabla_i\nabla_j\phi^\alpha + \Gamma^\alpha_{\beta\gamma}(\phi,\bar\phi)\partial_i\phi^\beta\partial_j\phi^\gamma\right) = 0\,, \tag{2.2}$$

and

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu}(\phi,\bar\phi)\,, \tag{2.3}$$

where the energy-momentum tensor of the moduli is

$$T_{\mu\nu} = -G_{\alpha\bar\beta}\left(\partial_\mu\phi^\alpha\partial_\nu\phi^{\bar\beta} - \frac{1}{2}g_{\mu\nu}\,g^{\rho\sigma}\partial_\rho\phi^\alpha\partial_\sigma\phi^{\bar\beta}\right). \tag{2.4}$$

It is useful to define $z \equiv (x_{D-2} + ix_{D-1})$ and rewrite the effective action as

$$S_{\rm eff} = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\,\sqrt{-g}\,R - \frac{1}{2\kappa^2}\int \mathrm{d}^{D-2}x\;E\,, \tag{2.5}$$

where $d^{D-2}x$ refers to the integration measure over the first $D-2$ coordinates, $x_0,\ldots,x_{D-3}$. E, which we later interpret as the energy density (tension) of the cosmic brane, is given by

$$E \equiv \int \mathrm{d}^2 z\sqrt{-g}G_{\alpha\bar\beta}g^{z\bar z}\left(\partial_z\phi^\alpha\partial_{\bar z}\phi^{\bar\beta} + \partial_{\bar z}\phi^\alpha\partial_z\phi^{\bar\beta}\right). \tag{2.6}$$

[2]In general the action contains a scalar potential, whose leading constant term represents an effective cosmological constant. Thus, in principle it is possible to discuss solutions which interpolate between Randall-Sundrum type of models and the solutions discussed below.

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We are interested in codimension-2 compactifications. Therefore we start with the following ansatz for the metric (writing $r \stackrel{\text{def}}{=} |z|$)

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + e^{2B(r)}\mathrm{d}z\mathrm{d}\bar{z}\,. \tag{2.7}$$

This type of ansatz has appeared recently in various field theory models [15, 16], supergravity inspired scenarios [26] and in the context of string theory [27, 28]. The authors of [26] considered the possibility of having a superpotential and hence a potential for the scalar fields. The scalars in our effective action are assumed to be true moduli, with no (super)potential.

In the remainder of this paper we will consider a single modulus scenario, $\phi^\alpha = \tau$ in which $\tau = a + ie^{-\Phi}$ is the axion-dilaton system of the $D = 10$ type-IIB string theory or compactifications to lower dimensions. The holomorphic solution $\tau = \tau(z)$ is that of D7-branes [25].

2.2 The spacetime metric

The general solution can be obtained along the lines of our earlier analysis [1]. Note that the diffeomorphism invariance can be used to reduce the number of integration constants in solving for the warp factors A and B:[3]

$$\begin{aligned} e^{2A(r)} &= [1 + a_0\log(r)]^{\frac{2}{D-2}}\,, & (2.8)\\ e^{2B(r)} &= l^2\,[1 + a_0\log(r)]^{-\frac{D-3}{D-2}}\,r^{-2-\xi[a_0\log(r)+2]}\,. & (2.9)\end{aligned}$$

We note that, in fact, both warp factors can be expressed in terms of the harmonic function $Z_7(r)$:

$$\begin{aligned} e^{2A(r)} &= [Z_7(r)]^{\frac{2}{D-2}}\,, \qquad [Z_7(r)] \stackrel{\text{def}}{=} [1 + a_0\log(r)]\,, & (2.10)\\ e^{2B(r)} &= l^2\,e^{\xi a_0^{-1}(1+1/\xi)^2}[Z_7(r)]^{-\frac{D-3}{D-2}}\,e^{-\xi a_0^{-1}[Z_7(r)+1/\xi]^2}\,, & (2.11)\end{aligned}$$

where the ξ-dependent factor appears akin to a screening factor, such as in the screened Coulomb potential. Here the constant l sets the length scale in the z-plane and is required on dimensional grounds, and ξ and a_0 are two integration constants obtained when solving the Einstein equations (2.3) with a non-zero stress tensor $T_{\mu\nu}$. In particular, ξ determines the Ricci tensor

$$R_{\mu\nu} = \mathrm{diag}[\underbrace{0,\ldots,0}_{D-1}, 2a_0\,\xi\,l^{-2}\,] \qquad \text{for } x^\mu = (t, x^1, \ldots, x^{D-3}, r, l\theta)\,, \tag{2.12}$$

where the polar angle has been rescaled as $l\theta$ for dimensional reasons. Through Einstein's equation, ξ then also determines the strength of the stress tensor:

$$\begin{aligned} [\,T_{\mu\nu}\,] &= [\,T(r)\,\eta_{ab}\,] \oplus \mathrm{diag}[\,-a_0\xi r^{-2}, a_0\xi\,]\,,\\ T(r) &= -a_0\,\xi\,l^{-2}\,[1 + a_0\log(r)]\,r^{\xi[2+a_0\log(r)]}\,. & (2.13)\end{aligned}$$

[3]We thank B. Kol for illuminating discussions on this point.

Furthermore, we note that ξ and a_0 are not entirely independent (see eq. (2.23) below):

$$\mathrm{sign}(\xi) = \mathrm{sign}(a_0)\,, \qquad \text{i.e.} \qquad \xi\, a_0 \geq 0\,. \tag{2.14}$$

In fact, since the Ricci 2-form, $R_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$, is a representative for the first Chern class, c_1, the result (2.12) together with (2.14) implies that $c_1 > 0$ when $\xi\, a_0 > 0$. As is well known, this obstructs supersymmetry. Now, although each of the three limits $a_0 \to 0$, $\xi \to 0$ and $l \to \infty$ removes this obstruction, we will see in section 2.6 that only the $a_0 \to 0$ limit leads to supersymmetric solutions.

For future reference, we note that the metric (2.7) with the warp factors (2.8) and (2.9) is almost invariant under the $r \to 1/r$ transformation. To be precise, we change variables $r \to 1/r$:

$$\begin{aligned} e^{2A(1/r)} &= [1 - a_0 \log(r)]^{\frac{2}{D-2}}\,, \\ e^{2B(1/r)} &= l^2\, [1 - a_0 \log(r)]^{\frac{-D-3}{D-2}}\, r^{-2+\xi[2-a_0\log(r)]}\,, \end{aligned} \tag{2.15}$$

where the r^{-2} factor in e^{2B} does not change, owing to the way it appears in the line element:

$$\mathrm{d}s^2 = \cdots + (\cdots r^{-2})(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2)\,, \tag{2.16}$$

and that $r^{-1}\mathrm{d}r$ and $\mathrm{d}\theta$ are invariant under the $r \to 1/r$ transformation. Thus, we have that the inversion transformation

$$\mathcal{I}\colon (r, a_0, \xi) \mapsto \left(\frac{1}{r}, -a_0, -\xi\right) \tag{2.17}$$

is a symmetry of the metric (2.7).

Finally, we note that the appearance of dimensionally unnormalized r as the argument of the logarithm in (2.8) and (2.9) indicates that we have set the radial length unit to 1.

Special properties of codimension-2. Before we proceed, a few remarks are in order regarding the special properties of codimension-2 solutions, i.e., when the transversal space is 2-dimensional.

First, note the logarithmic dependence of the warp factors (2.8) and (2.9). This, of course, is the expected behavior of Coulomb potentials, and (more generally) Green's functions of the laplacian in 2-dimensional space. Recall that in higher dimensions the corresponding quantities diverge at one end and vanish at the other end of the natural domain of the appropriately chosen radial variable, $r \in [0, \infty)$. This defines the location of the 'source', at $r = 0$, and the 'asymptotic region' at $r = \infty$. Unlike that, in 2 dimensions, the Green's functions diverge at both ends, $r = 0$ and $r = \infty$, implying that the 'source' is in this sense distributed at both of these locations. Note also that if the Coulomb potential (Green's function) is attractive (goes to negative infinity) at one end, it then is repulsive (goes to positive infinity) at the other end.

This dimensionally atypical behavior also implies the absence of an obvious candidate 'asymptotic region' at either end. Instead, we note that the logarithmic Coulomb potential necessarily vanishes at an intermediate radius. In our case (2.8) and (2.9), this is just $\log(r)$ and it vanishes at $r = 1$. For want of a better choice, we then treat the region around $r = 1$ as a surrogate for 'the asymptotic region'. Note however that, unlike in higher dimensions, the 'force' determined by a Coulomb potential in 2 dimensions does not vanish in the 'surrogate asymptotic region,' $r \sim 1$.

In addition, the harmonic function $Z_7(r)$, as defined in eq. (2.8), exhibits another special point, $r_s = e^{-1/a_0}$, where it vanishes. This is the location of a naked singularity; see in particular sections 2.4 and 2.5.

Having found a solution to the Einstein equations in the form (2.7), we now turn to a detailed discussion of the general solution for τ.

2.3 The general solution for the toral modulus

The equation of motion (2.2) for the toral modulus, $\tau = \tau(\theta)$, becomes:

$$\tau'' + \frac{2}{\bar{\tau} - \tau}(\tau')^2 = 0\,, \tag{2.18}$$

where the coefficient of $(\tau')^2$ is the Cristoffel connection, $\Gamma^{\tau}_{\tau\tau}$, derived from the Teichmüller metric; primes indicate derivatives with respect to the argument.

By writing $\tau = \tau_1 + i\tau_2$ and separating the real and the imaginary parts of eq. (2.18), we obtain the following system of two coupled, non-linear, second order ordinary differential equations:

$$\tau_2\tau_1'' = 2\tau_1'\tau_2'\,, \tag{2.19}$$
$$(\tau_1')^2 = (\tau_2')^2 - \tau_2\tau_2''\,. \tag{2.20}$$

From eq. (2.20), it is immediate that

$$\tau_1 = b_0 \pm \int \mathrm{d}\theta \sqrt{(\tau_2')^2 - \tau_2\tau_2''}\,. \tag{2.21}$$

Therefore, given a solution for $\tau_2(\theta)$, eq. (2.21) gives the corresponding $\tau_1(\theta)$.

2.3.1 Type-I solution

Here, we find the first branching point in the solution process. If the left-hand side of eq. (2.20) and so also the integrand in eq. (2.21) vanishes, the solution may be written in the form

$$\tau_I(\theta) = b_0 + i\, g_s^{-1}\, e^{\omega(\theta-\theta_0)}\,, \qquad b_0, g_s, \omega, \theta_0 = \text{const.}\,, \tag{2.22}$$

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and where g_s is the string coupling constant. Now, the Einstein equation (2.3) provides a relation between the constants in τ_I and those in the metric (2.7), with (2.8) and (2.9). In particular,

$$\omega = \sqrt{8\xi a_0}\,, \qquad \text{i.e.} \qquad \omega^2 = 8\xi a_0 \geq 0\,, \tag{2.23}$$

where the last inequality is enforced by the fact that ω must be real. Also, $e^{\omega\theta_0}$ can be reabsorbed in g_s^{-1}; equivalently, we set $\theta_0 = 0$ and fix the defining domain of θ to be $[-\pi, +\pi]$. Note furthermore that the constants b_0 and g_s are chosen such that rotations around the origin, i.e., varying θ through its defining domain induces an $\mathrm{SL}(2,\mathbb{Z})$ action on this solution:[4]

$$\tau_I(+\pi) = \frac{a\,\tau_I(-\pi) + b}{-\tau_I(-\pi) + d}\,. \tag{2.24}$$

Thus, in general only very special values of the string coupling will be allowed. We will return to this point in section 2.6 when we discuss the supersymmetric and F-theory limit of our solutions.

2.3.2 Type-II solution

On the other hand, if the left-hand side of eq. (2.20) does not vanish, this equation can be used to express $\tau_1(\theta)$ in terms of $\tau_2(\theta)$, as done in eq. (2.21). In this case we have:

$$\tau_{\mathrm{II}}(\theta) = b_0 \pm e^{\alpha_0}\frac{\sinh[\omega(\theta-\theta_0)] + i}{\cosh[\omega(\theta-\theta_0)]}\,, \qquad b_0, e^{\alpha_0}, \omega, \theta_0 = \text{const.} \tag{2.25}$$

We may again fix $\theta_0 = 0$ and choose the defining domain $\theta \in [-\pi, +\pi]$. As for τ_I, the Einstein equation (2.3) enforces the relation $\omega = \sqrt{8\xi a_0}$. Finally, since

$$\mathrm{Im}\, m(\tau_{\mathrm{II}}) = \pm e^{\alpha_0}\frac{i}{\cosh[\omega(\theta-\theta_0)]} = e^{-\Phi}\,, \tag{2.26}$$

we are forced to identify $e^{\alpha_0} = g_s^{-1}$, so

$$\tau_{\mathrm{II}}(\theta) = b_0 \pm g_s^{-1}\frac{\sinh[\omega(\theta-\theta_0)] + i}{\cosh[\omega(\theta-\theta_0)]}\,, \qquad b_0, g_s, \omega, \theta_0 = \text{const.} \tag{2.27}$$

Having exhausted all the logical possibilities in the solution process, we conclude that the two branches of solutions, $\tau_I(\theta)$ and $\tau_{\mathrm{II}}(\theta)$, as given in eqs. (2.22) and (2.27) respectively, represent the most general solutions to the modular equation of motion (2.18).

However, there is also the trivial solution, $\tau = \text{const.}$. Apart from satisfying (2.19)–(2.20), this solution can also be obtained as a special limit either of τ_I or

[4] By imposing a covariance under $\mathrm{SL}(2,\mathbb{Z})$, rather than $\mathrm{SL}(2,\mathbb{R})$, we expect our solution to be a string vacuum rather than just a (super)gravity solution.

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$\tau_{\rm II}$. When $g_s^{-1} \to 0$ both τ_I and $\tau_{\rm II}$ become equal to b_0, which follows from eqs. (2.22) and (2.27). In addition, by letting $\xi \to 0$, eq. (2.23) implies that τ_I, $\tau_{\rm II} \to b_0$.

Finally, we note that it is possible to choose b_0 and e^{α_0} such that rotations around the origin, i.e., varying θ through its defining domain induces a monodromy action on $\tau_{\rm II}(\theta)$:

$$\tau_{\rm II}(+\pi) = \tau_{\rm II}(-\pi) \pm n\,, \tag{2.28}$$

where n is an integer. We will return to the interpretation of n when discussing the supersymmetric limit in section 2.6.2.

It is remarkable that although both solutions, $\tau_I(\theta)$ and $\tau_{\rm II}(\theta)$, are strictly aperiodic functions of θ, it is nevertheless possible to ensure that they are $\mathrm{SL}(2,\mathbb{Z})$-covariant, as given in eqs. (2.24) and (2.28), respectively.

2.4 Spacetime properties

In the model considered above, where the dilaton and axion vary over $z = x_8 + ix_9$, the metric (2.7), with warp factors (2.8) and (2.9) and for $\xi = -1$, may be written as

$$\mathrm{d}s^2 = \left(\frac{\rho}{\rho_0}\right)^{\frac{2}{D-2}} \eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + l^2\left(\frac{\rho_0}{\rho}\right)^{\frac{D-3}{D-2}} e^{({\rho_0}^2-\rho^2)/\rho_0}\left(\mathrm{d}\rho^2 + \mathrm{d}\theta^2\right), \tag{2.29}$$

which is identical to that of the global cosmic brane solution studied by Cohen and Kaplan in [15]; we have used $\rho \stackrel{\rm def}{=} |a_0|^{-1}[1 - |a_0|\log(r)]$ and $\rho_0 \stackrel{\rm def}{=} |a_0|^{-1} = \rho|_{r=1}$, and l is the length scale introduced in eq. (2.9) for dimensional reasons.

In the remainder of this section we would like to study the spacetime properties of the general solution (2.8–2.9) in more detail. In particular, we want to answer questions about the stability of the solution and the relation to its supersymmetric cousins. To this end, we first look at the special case when $\xi = 0$.

2.4.1 The source-less case

The relation $\omega = \sqrt{8\xi a_0}$ then implies that also $\omega = 0$, so both $\tau_I(\theta)$ and $\tau_{\rm II}(\theta)$ become constant, and consequently the stress tensor, $T_{\mu\nu}$, now vanishes. The source-less Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 \tag{2.30}$$

are solved by the ansatz (2.7), with the warp factors (2.8) and (2.9) unchanged, except for setting $\xi = 0$:

$$e^{2A(r)} = [1 + a_0\log(r)]^{\frac{2}{D-2}}\,, \qquad e^{2B(r)} = l^2\,[1 + a_0\log(r)]^{-\frac{D-3}{D-2}}\,r^{-2}\,. \tag{2.31}$$

It is easy to check that $R = 0$ and hence $R_{\mu\nu} = 0$. Yet, the warp factors $e^{2A(r)}$ and $e^{2B(r)}$, respectively, vanish and diverge at $r_{\rm s} = e^{-1/a_0}$. In that sense, this solution is similar to the Schwarzschild solution.[5] Indeed, the quadratic curvature scalar

$$\mathrm{Riem}^2 \stackrel{\rm def}{=} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim a_0^4\,l^{-4}\,[1 + a_0\log(r)]^{-2\frac{D-1}{D-2}}\,, \tag{2.32}$$

[5]For a recent review see [29].

diverges at $r_{\rm s} = e^{-1/a_0}$. However, as opposed to the case of black holes, here the horizon[6] coincides with this singularity, which then is a naked (null) singularity.

In addition, at $r = 0$ for $a_0 < 0$ ($r = \infty$ for $a_0 > 0$) the Killing fields $\mathrm{d}/\mathrm{d}x^a$ along the brane diverge, i.e., there is a time-like singularity at the origin (infinity).[7] Furthermore, in the limit when $a_0 \to 0$ we recover a flat space solution though with nontrivial topology, $R^{1,8} \times S^1$. Therefore, it seems tempting to continue the analogy with the Schwarzschild solution, in which the physical singularity vanishes if the mass M of the black hole is taken to zero.

In order to explore this analogy further, consider the weak-field linearized Einstein equation [30, 31]:

$$\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, R\right)_{\rm weak-field} \approx \Box \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\, h\right) \stackrel{\rm def}{=} \Theta_{\mu\nu}, \tag{2.33}$$

where

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}\,, \qquad \text{and} \qquad h = \eta^{\mu\nu} h_{\mu\nu}\,. \tag{2.34}$$

Using the general form of our metric (2.7), we find that

$$h_{\mu\nu} = \mathrm{diag}\left[(1 - e^{2A}), \underbrace{(e^{2A} - 1), \dots, (e^{2A} - 1)}_{D-3}, (e^{2B} - 1), (e^{2B} - 1)\right], \tag{2.35}$$

and

$$h = (D-2)(e^{2A} - 1) + 2(e^{2B} - 1)\,, \qquad \text{for} \quad x^\mu = (t, x^1, \dots, x^{D-1})\,. \tag{2.36}$$

Since A and B depend only on r, we substitute $\Box \to \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} r \frac{\mathrm{d}}{\mathrm{d}r}$. This and the functional form of the warp factors (2.8) and (2.9) suggests using $\rho = a_0^{-1}[1 + a_0 \log(r)]$ as an abbreviation; then $\frac{\mathrm{d}}{\mathrm{d}\rho} = r\frac{\mathrm{d}}{\mathrm{d}r}$. The definition (2.33) then produces

$$\Theta_{tt} = \frac{1}{r^2}\left(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} e^{2B} + \frac{D-4}{2}\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} e^{2A}\right), \tag{2.37}$$

$$= \frac{4l^2}{r^4}[1 + a_0\log(r)]^{-\frac{D-3}{D-2}}\, C(r) - \left(\frac{D-4}{D-2}\right)^2 \frac{a_0^2}{r^2}[1 + a_0\log(r)]^{-2\frac{D-3}{D-2}}\,, \tag{2.38}$$

$$C(r) = 1 + \left(\frac{D-3}{D-2}\right) a_0[1+a_0\log(r)]^{-1} + \frac{(D-3)(2D-5)}{4(D-2)^2}\, a_0^2[1+a_0\log(r)]^{-2}\,. \tag{2.39}$$

The first term in (2.38) clearly diverges worse than the second, and for two different reasons. For one thing, the exponent of the logarithmic part of e^{2B} is less than that of e^{2A}. What is more important, however, e^{2B} contains a 'screening' factor, r^{-2}, which is the source of the r^{-4} factor in the first term of (2.38).

[6] Throughout this article, we will refer to what is more precisely called the 'putative horizon,' defined as the place where the lapse function (g_{tt}, for a diagonal metric) vanishes; see e.g. ref. [30, section 2.3].

[7] We thank D. Marolf for a discussion of this point.

Near $r = 1$, where the weak-field approximation ought to apply, we may neglect the logarithms, which leaves

$$\Theta_{tt} \sim \frac{4l^2}{r^4}\left[1 + \left(\frac{D-3}{D-2}\right)a_0 + \frac{(D-3)(2D-5)}{4(D-2)^2}\,a_0^2\right] - \left(\frac{D-4}{D-2}\right)^2 \frac{a_0^2}{r^2}\,. \tag{2.40}$$

In comparison, the energy-momentum "pseudo-tensor" for the Schwarzschild black hole solution is [31]

$$\Theta_{tt}^{BH} \sim \frac{M}{r^3}\,. \tag{2.41}$$

Thus, just as the black hole disappears when $M \to 0$, one might naively expect the black 7-brane solution to vanish when $a_0 \to 0$. This is in fact not quite the case, as eqs. (2.38)–(2.39) imply:

$$\lim_{a_0 \to 0} \Theta_{tt} \to \frac{4l^2}{r^4}\,. \tag{2.42}$$

This term can be traced to the 'background' source of r^{-2} in e^{2B}, indicating a (perhaps puzzling) non-trivial source. We wish to emphasize that this 'background' source is not unambiguously located at $r = 0$ only: the change of variables $r \to 1/r$ leaves $e^{2B}\mathrm{d}r^2$ invariant, except for the change $a_0 \to -a_0$. So, in a sense, this 'background' source is located both at $r = 0$ and at $r = \infty$.

To explore this further, we now include explicit sources in the analysis.

2.4.2 The case with sources

Let us consider a non-zero $T_{\mu\nu}$. The warp factors of the metric (2.8) and (2.9) read

$$\begin{aligned} e^{2A(r)} &= [1 + a_0 \log(r)]^{\frac{2}{D-2}}\,, \\ e^{2B(r)} &= l^2\,[1 + a_0 \log(r)]^{\frac{-D-3}{D-2}}\, r^{-2-\xi[2+a_0\log(r)]}\,, \end{aligned} \tag{2.43}$$

where we fix, for the time being, $a_0 \geq 0$. By taking the limit $\xi \to 0$, we recover the solution (2.31) to the source-free Einstein equations. Furthermore we compute the scalar curvature:

$$R \sim \xi\, a_0\, l^{-2}\, r^{\xi[2+a_0\log(r)]}\,[1 + a_0 \log(r)]^{\frac{D-3}{D-2}}\,, \tag{2.44}$$

the square of the Ricci tensor:

$$\mathrm{Ric}^2 \sim \xi^2\, a_0^2\; l^{-4} r^{2\xi[2+a_0\log(r)]}\,[1 + a_0 \log(r)]^{2\frac{D-3}{D-2}}\,, \tag{2.45}$$

and the square of the Riemann tensor:

$$\begin{aligned} \mathrm{Riem}^2 &\sim \mathcal{R}(r)\, a_0^4\, l^{-4}\, r^{2\xi[2+a_0\log(r)]}\,[1 + a_0 \log(r)]^{-2\frac{D-1}{D-2}}\,. \\ \mathcal{R}(r) &= \left(\frac{(D-1)(D-3)}{(D-2)^2} - 4\frac{(D-3)}{(D-2)}\xi a_0 \rho^2 + 4\frac{D}{(D-2)}\xi^2 a_0^2 \rho^4\right), \end{aligned} \tag{2.46}$$

where we again used the abbreviation $\rho \stackrel{\mathrm{def}}{=} a_0^{-1}[1 + a_0 \log(r)]$.

Note that Ric^2 and R provide no new information: Ric^2 behaves just like the ξ^2 term in Riem^2, and the scalar curvature simply behaves like $\sqrt{\mathrm{Ric}^2}$. Just as for the source-free solution, Riem^2 diverges at $r_{\mathrm{s}} = e^{-1/a_0}$. (Although non-zero in general, R and Ric^2 actually do vanish at r_{s}.)

In complete analogy with the source-less case, we compute the weak-field linearization of the Einstein tensor in this situation

$$\begin{aligned}\Theta^{\xi}_{tt} &= \frac{4l^2}{r^4}[1+a_0\log(r)]^{-\frac{D-3}{D-2}}\, r^{-\xi[2+a_0\log(r)]}\,\mathcal{C}_{\xi}(r) - \left(\frac{D-4}{D-2}\right)^2 \frac{a_0^2}{r^2}[1+a_0\log(r)]^{-2\frac{D-3}{D-2}}\,,\\ \mathcal{C}_{\xi}(r) &= \left[(1+\xi[1+a_0\log(r)])^2 + \frac{a_0\xi}{2}\right] + \\ &\quad + \left(\frac{D-3}{D-2}\right) a_0[1+a_0\log(r)]^{-1}(1+\xi[1+a_0\log(r)]) + \\ &\quad + \frac{(D-3)(2D-5)}{4(D-2)^2}\, a_0^2[1+a_0\log(r)]^{-2}\,. \end{aligned} \tag{2.47}$$

Again, in the weak-field regime (near $r = 1$), we neglect the logarithms, which leaves

$$\begin{aligned}\Theta^{\xi}_{tt} &\sim \frac{4l^2}{r^4} r^{-2\xi}\,\mathcal{C}_{\xi}(1) - \left(\frac{D-4}{D-2}\right)^2 \frac{a_0^2}{r^2}\,,\\ \mathcal{C}_{\xi}(1) &= \left[(1+\xi)^2 + \frac{a_0\xi}{2}\right] + \left(\frac{D-3}{D-2}\right) a_0(1+\xi) + \frac{(D-3)(2D-5)}{4(D-2)^2}\, a_0^2\,. \end{aligned} \tag{2.48}$$

From this, we furthermore find

$$\lim_{a_0\to 0+} \Theta^{\xi}_{tt} \sim 4l^2\,(1+\xi)^2\, r^{-2(1+\xi)-2}\,, \tag{2.49}$$

which is to be compared with the corresponding expression for the supersymmetric D7-branes of refs. [27]. In fact, the metric (2.7) can be specialized so as to correspond to the result of refs. [27], by setting

$$e^{2A} \to 1\,, \qquad \text{and} \qquad e^{2B} \to [1+\lambda\log(r)]\, r^{2B_1}\,. \tag{2.50}$$

In addition, $\tau \to -\lambda + i[1+\lambda\log(r)]$. From these, we compute

$$\Theta_{tt} = -\lambda\,\delta(r)\, r^{2B_1} + 4B_1\lambda r^{2B_1-2} + 4B_1^2[1+\lambda\log(r)]\, r^{2B_1-2}\,. \tag{2.51}$$

Thus,

$$\lim_{\lambda\to 0} \Theta_{tt} \sim 4B_1^2\, r^{2B_1-2}\,, \tag{2.52}$$

which agrees with (2.49) upon the identification $B_1 = -(\xi+1)$ when $a_0, \xi \geq 0$.

The analogous result for the $a_0 \leq 0$ case is now easy to deduce, owing to the inversion symmetry (2.17). We now obtain

$$\lim_{a_0\to 0-} \Theta^{-\xi}_{tt} \sim 4l^2\,(1-\xi)^2\, r^{-2(1-\xi)-2}\,, \tag{2.53}$$

and therefore $B_1 = -(1+|\xi|)$ in general.

On one hand, our a_0 and the λ in refs. [27] appear in an analogous, but not identical, fashion in the metric. In addition, the functional dependence of both the axion and the dilaton in our configurations is different from that in refs. [27]. However, our solutions agree with that of refs. [27] only in the simultaneous limit $a_0, \lambda \to 0$. Also, comparison with ref. [23] shows that this indicates (in the present limit) the presence of $12(1+|\xi|)$ supersymmetric 7-branes. The extra 'background' sources, remaining in the $\xi \to 0$ limit, have already shown up in our calculations for the source-less case, in eq. (2.42). In this sense, the source-less solution is akin to the Schwarzschild metric, where there is a singularity without a corresponding 'matter' ($T_{\mu\nu}$) source.

This analysis indicates that, of the limits discussed in section 2.2, it is the $a_0 \to 0$ limit that makes our solutions supersymmetric. We will discuss this in more detail in section 2.6.

2.4.3 The nature of the naked singularity

While having a naked singularity is not desirable from a purely classical point of view, it is possible that the singularity may be resolved in a string theory context.[8] Recently, examples of so called "repulsons", a repulsive naked singularity [32], have been studied in supergravity. These were found to be harmless once they were considered in the dual gauge theory formulation [33]–[37] — this is the enhançon mechanism. It is therefore important to consider the classical nature of the above naked singularity as a first step towards answering the question whether there exists a non-supersymmetric effect analogous to the enhançon mechanism in our case.

Let us compute the time it takes a test particle to fall into, or get reflected by the naked singularity. Following Kallosh and Linde [32] (see also [15, 38] and [39]) we have for a particle with mass m, energy E, and angular momentum L,

$$t = E \int \mathrm{d}r \frac{\sqrt{g_{rr}}}{g_{00}} \left(\frac{E^2}{g_{00}} - g_{rr}^{-1} \frac{L^2}{r^2} - m^2 \right)^{-1/2} . \tag{2.54}$$

As discussed in section 2.2, the only reasonable candidate for an asymptotic region is around $r = 1$. This is where the quantities E, L, m need to be evaluated for the equation (2.54).

To decide whether the singularity is repulsive or attractive (and hence whether spacetime is geodesically complete or not) we have to see if the denominator of the integrand has a zero. This would correspond to the test particle coming to a stop and hence getting reflected from the singularity. For simplicity let us set $L = 0$. The answer only depends on the behavior of g_{00} close to the singularity. Since $g_{00} \to 0$, the particle does not turn around. From this equation, one sees that

[8] A simple example of that is the supersymmetric 7-brane in supergravity which has a naked singularity at a finite distance away from the origin. However, this UV-problem is resolved when considering the D7-brane in string theory, because of the $\mathrm{SL}(2,\mathbb{Z})$ covariance.

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the time the test particle takes to reach the naked singularity is finite. Hence our solution is geodesically incomplete and the effective potential is attractive. Note that the Schwarzschild solution shares the same features with our solution.

However, a quantum mechanical treatment of this problem shows that the potential is so attractive that there are no normalizable bound states. Hence, the particle must be reflected, i.e., remain a superposition of scattering states [15, 1]. The explicit treatment of a quantum scalar particle probe of our background was done in Appendix A of [1], and will not be repeated here. Suffice it here to say that the wave functions of the probe vanish as the singularities are approached. Thus, effectively, the whole spacetime looks like a box with impenetrable walls at the positions of the singularities. Finally, although we used a scalar probe in ref. [1], the analysis there suggests that this qualitative feature is independent of the scalar nature of the probe. Herein, we will assume this to be true.

2.5 The transverse space

In order to understand the effects of the charged 7-branes in our solution it is important to study their effect on the transverse, z-space. We first note that the longitudinal part of spacetime, spanned by the coordinates $t, x^1, \dots, x^{D-3}$, is fibered over the transverse space. For any point on the base the fiber is just a $(D-2)$-dimensional Minkowski space. However, because of the behavior of the warp factor e^{2A} in the metric (2.7), the distances within this $(D-2)$-dimensional Minkowski space vary over the transverse, z-space. In particular, all distances and in fact the volume of the entire $(D-2)$-dimensional Minkowski space vanishes at the naked singularity. When we wrap this solution on a compact space, such as K3, we will see that although the supergravity solution breaks down when $V_{K3} \to 0$, string theory is still well-defined.

2.5.1 Physically permissible regions

Owing to the appearance of the logarithm and the fractional powers in the warp factors (2.8) and (2.9), not all values of $r \in [0, +\infty)$ are physically admissible. In fact, depending on the sign of a_0, there appear two physically permissible regions of the transverse space.

First of all, the naked singularity occurs at $r_s^+ \stackrel{\text{def}}{=} e^{-1/a_0} = e^{-1/|a_0|}$ for $a_0 > 0$, but at $r_s^- \stackrel{\text{def}}{=} e^{-1/a_0} = e^{+1/|a_0|}$ when $a_0 < 0$. It should be clear that

$$r_s^+ < r_s^- \qquad \text{for } |a_0| < \infty\,, \qquad \text{and} \qquad \lim_{a_0 \to +\infty} r_s^+ = 1 = \lim_{a_0 \to -\infty} r_s^-\,. \tag{2.55}$$

For $a_0 > 0$, the warp factors, e^{2A} and e^{2B} and so also the metric (2.7) are all finite, real and positive as long as $r \in [r_s^+, \infty)$. However, for $r < r_s^+$ the warp factors become complex: $e^{2A} = |e^{2A}| e^{\frac{2i\pi}{D-2}}$ and $e^{2B} = |e^{2B}| e^{-\frac{D-3}{D-2} i\pi}$. Therefore, when $a_0 > 0$ only $r_s^+ \le r < \infty$ is physical. We refer to this branch as the 'outside' region of the transverse space.

On the other hand, when $a_0 < 0$, the warp factors, e^{2A} and e^{2B}, and so also the metric (2.7) are all finite, real and positive while $r \in (0, r_s^-]$. However, the warp factors now become complex ($e^{2A} = |e^{2A}| e^{\frac{2i\pi}{D-2}}$ and $e^{2B} = |e^{2B}| e^{-\frac{D-3}{D-2} i\pi}$) for $r > r_s^-$. Therefore, when $a_0 < 0$ only $0 < r \leq r_s^-$ is physical. This complementary branch is naturally identified as the 'inside' region of the transverse space.

Clearly, the inversion symmetry (2.17) maps the 'outside' and the the 'inside' region into one another. Both of these regions have the topology of $\mathbb{R}_+ \times S^1$, containing the boundary circle with $r = r_s^\pm$. (To be pedantic, the 'inside' region of the transverse space is $\mathbb{R}_- \times S^1$, but topologically $\mathbb{R}_+ \cong \mathbb{R}_-$.) Owing to the relations (2.55), the two regions overlap in the annulus $r_s^+ \leq r \leq r_s^-$. We may then consider a candidate transverse space manifold obtained by gluing the two regions together into an 'open' annulus: the z-plane with the origin excised. This has the topology of $\mathbb{R}^1 \times S^1$.

In this article, we focus on the two separate physically permissible regions of the transverse space: 'outside' with $a_0 > 0$ and $r \in [r_s^+, \infty)$, and 'inside' with $a_0 < 0$ and $r \in (0, r_s^-]$, and their respective $a_0 \to 0$ limits.

2.5.2 The source-less case

Again, we start by examining the source-less case, and focus on the transverse part of the metric. In terms of the standard polar coordinates we have,

$$\mathrm{d}s_\perp^2 = l^2 \left[1 + a_0 \log(r)\right]^{-\frac{(D-3)}{(D-2)}} r^{-2} (\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2)\,. \tag{2.56}$$

In writing the metric in this form we have used the diffeomorphism invariance for the longitudinal part (which removes all independent integration constants for e^{2A} and fixes the overall scaling factor for e^{2B}). Thus, we are left with one free parameter, a_0, which can be either positive or negative.

The transverse metric can be rewritten in the standard form,

$$\mathrm{d}s_\perp^2 = \mathrm{d}p^2 + P(p)^2 \mathrm{d}\theta^2\,, \tag{2.57}$$

so that

$$\mathrm{d}p = l \left[1 + a_0 \log(r)\right]^{-\frac{(D-3)}{2(D-2)}} r^{-1} \mathrm{d}r\,. \tag{2.58}$$

When integrating this equation, it remains to determine the lower limit of the r-integration on the left hand side. Since the natural boundary of r is e^{-1/a_0} (that is, r_s^+ in the 'outside' region, and r_s^- in the 'inside' region), it seems reasonable to use this as the reference point when integrating the right hand side of (2.58). However, with the benefit of hindsight, we will leave this lower limit, r_0, unspecified for now and introduce a corresponding constant. With this in mind, we find the proper distance to be

$$p = \tilde{p}_0 + \frac{2l}{a_0} \frac{D-2}{D-1} \left(\left[1 + a_0 \log(r)\right]^{\frac{D-1}{2(D-2)}} - \tilde{\lambda} \right), \tag{2.59}$$

where the constants in the p- and r-integration are $\tilde{p}_0$ and $\tilde{\lambda}$, respectively. In fact, since we restrict r to the physically permissible region, $[r_s^+, +\infty)$ when $a_0 > 0$ and $(0, r_s^-]$ when $a_0 < 0$,

$$\tilde{\lambda} \stackrel{\text{def}}{=} [1 + a_0 \log(r_0)]^{\frac{D-1}{2(D-2)}} \;\Rightarrow\; \tilde{\lambda} \geq 0\,. \tag{2.60}$$

The naked singularity is now located at

$$p_s \stackrel{\text{def}}{=} p(r_s) = \tilde{p}_0 - \frac{2l\tilde{\lambda}}{a_0}\frac{D-2}{D-1}\,. \tag{2.61}$$

Depending on the sign of a_0, the qualitative features of $p(r)$ vary. We discuss in turn these two different proper distances. In particular, note that

$$p_s^+ = \tilde{p}_0 - \frac{2l\tilde{\lambda}}{|a_0|}\frac{D-2}{D-1} < p_s^- = \tilde{p}_0 + \frac{2l\tilde{\lambda}}{|a_0|}\frac{D-2}{D-1}\,. \tag{2.62}$$

It should be clear that while $\tilde{p}_0$ shifts the positions of p_s^+ and p_s^- *together*, $\tilde{\lambda}$ controls the splitting between p_s^+ and p_s^-. As long as we discuss either one of the two regions of the transverse space, using two constants is redundant and we may fix $\tilde{p}_0 = 0$; $\tilde{\lambda}$ then controls $p^\pm$, the position of the naked singularity.

For $a_0 > 0$, $p = p^+(r)$ is real, and varies as $p^+(r) \in [\tilde{p}_0, +\infty)$ when $r \in [r_s = e^{-1/a_0}, \infty)$, i.e., in the 'outside' physical region. However, in the unphysical region $r < r_s$, the proper distance $p^+(r)$ becomes complex and of the form $|p^+|e^{i\frac{D-1}{2(D-2)}\pi}$, just as do the warp factors and the whole metric (2.7). This then further justifies our assessment that $0 < r < r_s^+$ is unphysical when $a_0 > 0$. Note that this 'outside' region of the transverse space is parametrized by $p^+(r) \in [p_s^+, +\infty)$, and so includes the boundary circle at $p = p_s^+$, where the naked singularity is located.

On the other hand, when $a_0 < 0$, $p = p^-(r)$ is real, and varies as $p^-(r) \in (-\infty, \tilde{p}_0]$ when $r \in (0, r_s = e^{-1/a_0}]$, i.e., in the 'inside' physical region. However, for $r > r_s$ the proper distance function $p^-(r)$ becomes complex of the form $|p^-|e^{i\frac{D-1}{2(D-2)}\pi}$, just as do the warp factors and the metric. This again justifies our earlier conclusion that $r_s^- < r < \infty$ is unphysical when $a_0 < 0$. This 'inside' region of the transverse space is parametrized by $p^-(r) \in (-\infty, p_s^-]$, and so includes the location of the naked singularity: the boundary circle at $p = p_s^-$.

Since $\tilde{\lambda} \geq 0$, the two branches overlap for $p_s^+ < p < p_s^-$. Then, for suitably chosen $\tilde{\lambda}$, the two functions $p^\pm(r)$ will have one or two common points (see figure 1). In fact, for $\tilde{\lambda} = 1$, the two proper distances not only meet at $r = 1$ (with $p^+(1) = \tilde{p}_0 = p^-(1)$), but are in fact tangential to each other at that point.

It is then possible to construct a manifold by gluing the two patches together along the 'unit' circle ($r = 1$ and $p = \tilde{p}_0$). This new transverse space would then be parametrized by $p^- \in (-\infty, \tilde{p}_0]$ and $p^+ \in [\tilde{p}_0, +\infty)$. Topologically, this is a cylinder, $\mathbb{R}^1 \times S^1$. Owing to the smooth joining of p^+ and p^- at $r = 1$, the mapping from the

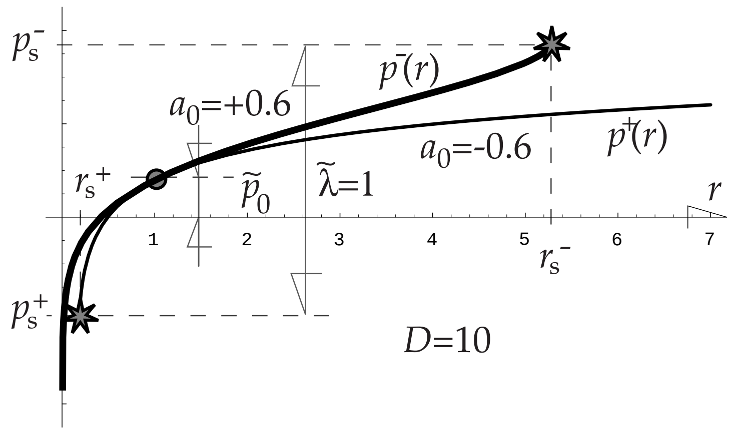

Figure 1: A plot of the two branches of the proper distance, $p^+(r)$ and $p^-(r)$, as functions of the (flat) auxiliary distance, r.

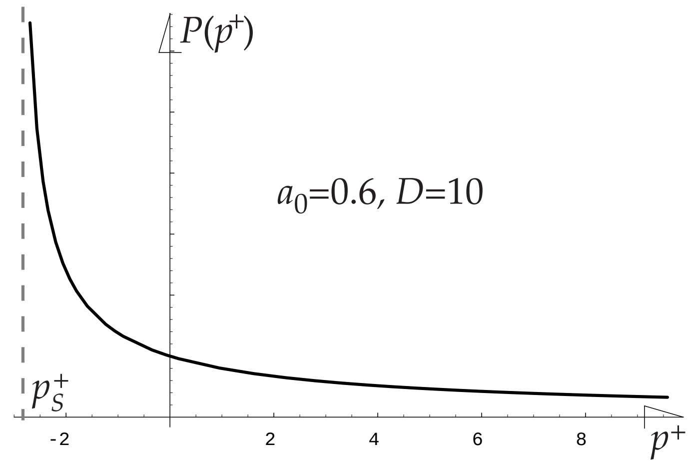

Figure 2: A plot of the distance function $P(p^+)$, for the source-less case, as a function of the proper distance, p^+. Note that $P(p^-) = P(2\tilde{p}_0 - p^+)$, so that $P(p^-)$ is a reflection of $P(p^+)$ and the two plots both equal 1 at $\tilde{p}_0$.

(r,θ)-plane (minus the origin) to the (p,θ)-cylinder is not only continuous but also smooth.

Next, we compute

$$P(p) = l\left(\tilde{\lambda} + \frac{a_0}{2l}\frac{D-1}{D-2}(p-\tilde{p}_0)\right)^{-\frac{D-3}{D-1}} . \tag{2.63}$$

Note that at the naked singularity, $p_\mathrm{s} = \tilde{p}_0 - \frac{2l\tilde{\lambda}}{a_0}\frac{D-2}{D-1}$ and $P(p_\mathrm{s}) = \infty$; see figure (2). On the other end, $P(\pm\infty) = 0$.

Finally, we compute the deficit angle at infinity

$$\Delta = 2\pi\left(1 - \lim_{p\to\infty}\frac{P(p)}{p}\right) = 2\pi\,. \tag{2.64}$$

Therefore the geometry of the transverse space is cylindrical, with a varying radius $P(p)$. When $a_0 > 0$ the cylinder is pinched at $p = \infty$, i.e., its circumference, $2\pi P(p)$, rather quickly diminishes to zero. On the other end, $P(p)$ diverges at p_s^+: the circumference becomes infinitely big. Thus, somewhat counterintuitively, the proper distance, p^+, in the 'outside' region of the transverse space grows from the circle of infinite circumference at p_s^+, toward circles of vanishing circumferences, at $p^+ \to \infty$.

Similarly, for $a_0 < 0$ the cylinder is pinched at $p = -\infty$, where the circumference shrinks to a point. Also, $P(p)$ diverges at p_s^-, rendering the circumference infinitely big. In contrast to the 'outside' region region, in the 'inside,' the proper distance, p^-, grows from the circles of vanishing circumference at $p^- \to -\infty$, toward the circle of infinite circumference, at $p^- = p_s^-$.

The (linear) curvature scalar, $R_\perp$, of the transverse space is:

$$R_\perp = -\frac{D-3}{D-2}\, a_0^2\, l^{-2}[1 + a_0 \log(r)]^{-\frac{D-1}{D-2}}\,. \tag{2.65}$$

The limit $a_0 \to 0$ is clearly interesting, since then this (invariant) curvature of the transverse space vanishes, and so the transverse space becomes flat. This justifies our earlier conclusions involving the weak-field linearized Einstein tensor $\Theta_{\mu\nu}$ in eq. (2.39). In addition, it is straightforward to see that

$$\lim_{a_0 \to 0} P(p) = l\tilde{\lambda}^{-\frac{D-3}{D-1}}\,, \tag{2.66}$$

i.e., the transverse space attains the shape of a flat cylinder of constant circumference $2\pi l\tilde{\lambda}^{-\frac{D-3}{D-1}}$. This result finally gives a transparent physical interpretation of this constant. Furthermore, from the expression for p_S we see that the naked singularity is pushed to $p_s^+ \to -\infty$ for $a_0 \to +0$ ($p_s^- \to +\infty$ for $a_0 \to +0$). Thus,

$$\lim_{a_0 \to 0+} p^+ \in (-\infty, +\infty)\,, \qquad \text{and} \qquad \lim_{a_0 \to 0-} p^- \in (-\infty, +\infty)\,. \tag{2.67}$$

That is, both the 'outside' and the 'inside' region now extend over the whole cylinder.

Following the standard procedure for T-duality (see e.g [36]), we dualize on the θ coordinate. The only element in the T-dual metric that is different from the original one (2.7) is

$$\tilde{g}_{\theta\theta} \stackrel{\text{def}}{=} \frac{1}{g_{\theta\theta}} = \frac{\tilde{l}^2}{\alpha'}[1 + a_0 \log(r)]^{\frac{D-3}{D-2}}\,. \tag{2.68}$$

Here, $\tilde{l} = \alpha'\, l^{-1}$. Of course, the dilaton now becomes non-constant:

$$e^{2\tilde{\Phi}} = \frac{\tilde{l}^2}{\alpha'} g_s^2 [1 + a_0 \log(r)]^{\frac{D-3}{(D-2)}}\,. \tag{2.69}$$

Very near the naked singularity where $r \sim r_s$, $e^{2\tilde{\Phi}} \sim g_s^2 \frac{\tilde{l}^2}{\alpha'}(r - r_s)$.

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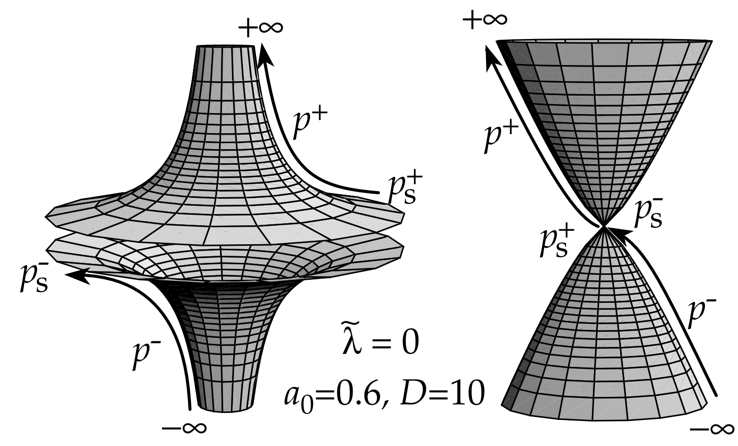

Figure 3: A plot of the two regions of the transverse space (left) and their θ-dual (right). The plot on the right is clipped at finite radius (given by $P(p^{\pm})$; the two regions join at the naked singularity, at $P(p_s^+) = \infty = P(p_s^-)$. For $\tilde{\lambda} < 0$, the two regions, and also their duals, separate; for $\tilde{\lambda} > 0$ they intersect at a finite value of $P(p^{\pm})$, i.e., $\tilde{P}(p^{\pm})$.

In this dual picture, all the previous results, from (2.58) to just before (2.63) still apply, but eq. (2.63) is now substituted by

$$\tilde{P}(p) = \tilde{l}\left[\tilde{\lambda} + \frac{a_0}{2l}\frac{D-1}{D-2}(p - \tilde{p}_0)\right]^{\frac{D-3}{D-1}} . \tag{2.70}$$

Note that at the naked singularity, $p_s = \tilde{p}_0 - \frac{2l\tilde{\lambda}}{a_0}\frac{D-2}{D-1}$ and $\tilde{P}(p_s) = 0$. On the other end, $P(+\infty) = \infty$ for $a_0 > 0$, and $P(-\infty) = \infty$ for $a_0 < 0$.

We again compute the deficit angle at infinity

$$\Delta = 2\pi\left(1 - \lim_{p\to\infty}\frac{\tilde{P}(p)}{p}\right) = 2\pi\,. \tag{2.71}$$

Therefore the geometry of the transverse space is still cylindrical, with a varying radius $\tilde{P}(p)$. This time, however, for both $a_0 > 0$ and $a_0 < 0$ the cylinder is pinched at $p = p_s^{\pm}$, i.e., its circumference, $2\pi\tilde{P}(p)$, rather quickly diminishes to zero near the naked singularity. On the other end, $\tilde{P}(p)$ diverges as $p^{\pm} \to \pm\infty$: the circumference becomes infinitely big. This time, the proper distance, p^+, in the 'outside' region of the transverse space grows from the circle of vanishing circumference at p_s^+, toward circles of unbounded circumferences, at $p^+ \to \infty$. Again in contrast to the 'outside' region, the 'inside' proper distance, p^-, grows from the circles of unbounded circumference at $p^- \to -\infty$, toward the circle of vanishing circumference, at $p^- = p_s^-$. Qualitatively, this image (with the 'inside' and 'outside' regions patched at the naked singularity) is a two-sheeted cone, see figure 3. We may thus understand the configurations discussed here and in ref. [1] as the θ-duals of a two-sheeted cone.

In addition, it is straightforward to see that in the $a_0 \to 0$ limit,

$$\lim_{a_0 \to 0} \tilde{P}(p) = \tilde{l}\tilde{\lambda}^{\frac{D-3}{D-1}}\,, \tag{2.72}$$

i.e., the transverse space attains the shape of a flat cylinder of constant circumference $2\pi l\tilde{\lambda}^{\frac{D-3}{D-1}}$. In the radial direction, we still have

$$\lim_{a_0 \to 0+} p^+ \in (-\infty, +\infty)\,, \qquad \text{and} \qquad \lim_{a_0 \to 0-} p^- \in (-\infty, +\infty)\,. \tag{2.73}$$

That is, both the 'outside' and the 'inside' region now extend over the whole cylinder.

2.5.3 The case with sources

Next we re-introduce sources in the Einstein equations. The transverse metric is then

$$\mathrm{d}s_\perp^2 = l^2[1 + a_0\log(r)]^{\frac{-(D-3)}{(D-2)}} r^{-2-\xi[2+a_0\log(r)]}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2). \tag{2.74}$$

We repeat the analysis above and now find that

$$p - p_0 = \frac{l}{2a_0}\, e^{\xi/2a_0} \left(\frac{2a_0}{\xi}\right)^{\frac{D-1}{4(D-2)}} \int_{t(r_0)}^{t(r)} \mathrm{d}t t^{\frac{D-1}{4(D-2)}-1} e^{-t}\,, \tag{2.75}$$

where we introduced $t \stackrel{\text{def}}{=} \frac{\xi}{2a_0}[1 + a_0\log(r)]^2$. The integral on the right hand side produces the incomplete γ-function:

$$p = p_0 + \frac{l}{2a_0}\, e^{\xi/2a_0} \left(\frac{2a_0}{\xi}\right)^{\frac{D-1}{4(D-2)}} \left[\gamma\left(\frac{D-1}{4(D-2)}; \frac{\xi}{2a_0}[1 + a_0\log(r)]^2\right) - \lambda\right]. \tag{2.76}$$

As before, we have restricted r to the physically permissible region, $[r_{\mathrm{s}}^+, +\infty)$ when $a_0 > 0$ and $(0, r_{\mathrm{s}}^-]$ when $a_0 < 0$. Therefore, with $r_0 \in [r_{\mathrm{s}}^+, +\infty)$,

$$\lambda \stackrel{\text{def}}{=} \gamma\left(\frac{D-1}{4(D-2)}; \frac{\xi}{2a_0}[1 + a_0\log(r_0)]^2\right) \Rightarrow \lambda \geq 0\,. \tag{2.77}$$

The resulting expression is impossible to invert in closed form, and so we are unable to express $P(p)$ in closed form, although it is of course readily given in terms of the auxiliary (flat) r coordinate:

$$P(p(r)) = l\,[1 + a_0\log(r)]^{\frac{-(D-3)}{2(D-2)}} r^{-\xi[1+\frac{1}{2}a_0\log(r)]}\,. \tag{2.78}$$

Also, figure 4 presents a plot, for three sample values of ξ, of $P(p^+(r))$ vs. $p^+(r)$.

However, note that $P(p) = 0$ now occurs for finite p,

$$p_{\max} = p_0 + \frac{l}{2a_0}\, e^{\frac{\xi}{2a_0}} \left(\frac{2a_0}{\xi}\right)^{\frac{D-1}{4(D-2)}} \Gamma\left(\frac{D-1}{4(D-2)}\right), \tag{2.79}$$

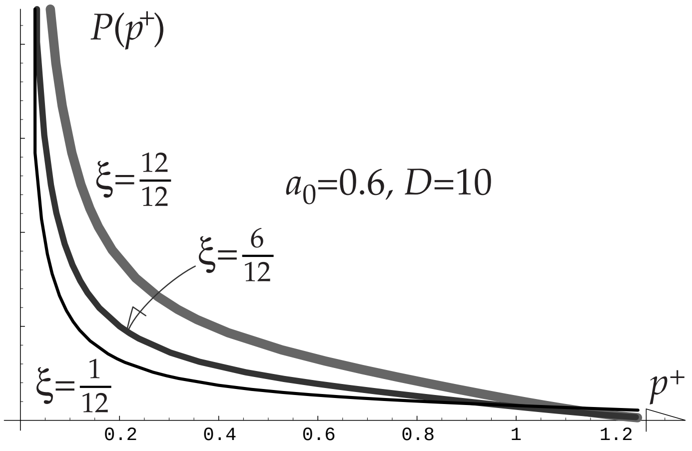

Figure 4: A (parametric) plot of the distance function $P(p^{+}(r))$ as a function of the proper distance, $p^{+}(r)$, plotted for the case with sources, for a few select values of ξ.

which has the correct limit $p_{\max} \to \infty$ when $\xi \to 0$, for the source-less case. Other than that, $P(p^{+})$ behaves the same for all $0 \leq \xi \leq 1$.

The (linear) curvature scalar, $R_{\perp}$, of the transverse space now becomes:

$$R_{\perp} = -\left[\frac{D-3}{D-2} + 2(\xi a_0^{-1})[1 + a_0 \log(r)]^2\right] a_0^2\, l^{-2}[1 + a_0 \log(r)]^{-\frac{D-1}{D-2}}\, r^{+\xi[2+a_0 \log(r)]}\,. \tag{2.80}$$

There are two interesting limits we can consider: $\xi \to 0$, to recover the source-less case, and $a_0 \to 0$ (while keeping ξ constant), which renders the transverse space flat.

Indeed, in the limit $\xi \to 0$, the scalar curvature (2.80) does agree with the source-less result (2.65), as expected. Note also that both of these curvatures vanish as $a_0 \to 0$, and ξ is held constant in (2.80). As in the source-less case, the naked singularity is gone in the $a_0 \to 0$ limit.

Let us now turn to discuss the $a_0 \to 0$ limit. In particular, we wish to determine how the presence of sources ($\xi \neq 0$) affects the dependence of the length of the circumference, $2\pi P(p)$, on the proper distance, p. Although there is no expression $P(p)$ in closed form for arbitrary a_0, in the $a_0 \to 0$ limit, the expression for $P(p)$ simplifies to

$$P(r)|_{a_0=0} = r^{-\xi}\,. \tag{2.81}$$

Furthermore, when $a_0 \to 0$ the expression for $p(r)$ simplifies into

$$p - p_0 = -\frac{l}{\xi} r^{-\xi}\,. \tag{2.82}$$

Hence, eq. (2.81) becomes

$$P(p)|_{a_0=0} = l\xi(p_0 - p)\,. \tag{2.83}$$

This is to be compared with the result (2.66) for the source-less case. For positive ξ, and so also positive $a_0 \to 0+$ (recall the relations (2.14)) we are in the 'outside' region, and eq. (2.81) clearly indicates that the circumference of the cylindrical transverse space, $2\pi P(p^+) \sim 2\pi r^{-|\xi|}$ is shrinking with a growing p^+; thus, the transverse space is pinching at $p^+ \to \infty$.

When ξ is negative, $a_0 \to 0-$, and we are in the 'inside' region. Recall that now the proper distance, $p^- \to -\infty$ when $r \to 0$. The circumference, $2\pi P(p^-) \sim 2\pi r^{|\xi|}$, of the cylindrical transverse space again pinches at $p^- \to -\infty$. The two regions of the transverse space thus turn out to have very similar geometries.

It is now straightforward to compute the deficit angle:

$$\Delta = 2\pi \left(1 - \lim_{p\to\infty} \frac{P(p)}{p} \right) = 2\pi(1+\xi)\,. \tag{2.84}$$

Thus the choice $a_0 \geq 0$, in which case $\xi > 0$, gives us the standard D7-branes in the limit $a_0 \to 0$, while $a_0 \leq 0$ formally implies that the number of branes is negative.

However, recall the special property of the codimension-2 solutions, that the sources of Coulomb potentials are located at both $r = 0$ and $r = \infty$. To check this for the case $a_0 \to 0-$ (for $a_0 \leq 0$), we change coordinates $r \to 1/r$. The distance function is then given by, for $a_0 < 0$,

$$P(p)|_{a_0=0} = l|\xi|(p_0 - p)\,, \tag{2.85}$$

and hence the deficit angle becomes

$$\Delta = 2\pi(1 + |\xi|)\,. \tag{2.86}$$

The conclusion is then that for $a_0 \to 0+$ the 7-branes are located at $r = 0$ while for $a_0 \to 0-$ the 7-branes are located at $r = \infty$. As we will see in section 4 this is consistent with the resolution of the naked singularity when the 7-brane is wrapped on a K3.

2.6 Supersymmetric limit and F-theory

2.6.1 Supersymmetry

With the above identification of our brane solution we now turn to verify that the $a_0 \to 0$ limit indeed gives rise to a supersymmetric vacuum.

In order to establish that there exists a supersymmetric limit of our solution let us study the harmonic function $Z_7(r)$ which figures in the expression for the metric (2.8)–(2.9). For the metric (2.7), the condition for having a Killing spinor (i.e., the supersymmetry variation of the gravitino) is that the first warp factor, e^{2A}, is constant[9] (see eg. [40]). That is, $A'(r)$ is the obstruction for supersymmetry. From

$$\begin{aligned} e^{2A(r)} &= [1 + a_0 \log(r)]^{\frac{2}{D-2}} \stackrel{\text{def}}{=} Z_7^{\frac{2}{D-2}} \,, \\ A'(r) &= \frac{1}{(D-2)} \frac{a_0}{r(1 + a_0 \log(r))} \,, \end{aligned} \tag{2.87}$$

we have $Z_7(r) \to 1$ and $A'(r) \to 0$ when $a_0 \to 0$. Thus the obstruction to supersymmetry is lifted. While the first Chern class of the spacetime does vanish in both of the other two limits, $\xi \to 0$ and $l \to \infty$ (see section 2.2), they do not affect the (non)vanishing of the supersymmetry obstruction, $A'(r)$. This unambiguously identifies $a_0 \to 0$ as the supersymmetric limit.

As further evidence let us now consider whether the solution saturates the BPS bound:

$$E \geq \Delta \,. \tag{2.88}$$

In section 2.5 we found that there exist two distinct contributions to the deficit angle, Δ, when $\xi \neq 0$. There is on one hand the purely gravitational, or background part, $\Delta_0 = 2\pi$, from eq. (2.64). In addition, we have $\Delta_\xi = 2\pi|\xi|$ from the stress-tensor, $T_{\mu\nu}$. Thus, the total deficit angle is then

$$\Delta = \Delta_0 + \Delta_\xi = 2\pi(1 + |\xi|) \,. \tag{2.89}$$

We now turn to the tension, E. While it was possible to identify the gravitational and stress tensor contributions to the deficit angle separately, we only have a means of computing the tension due to the non-zero $T_{\mu\nu}$. The tension defined by (2.6) is

$$\begin{aligned} E_\xi &= 2\pi \frac{\omega^2}{8a_0} \int_{r_s^+}^{\infty} \frac{\mathrm{d}r}{r} 2a_0 [1 + a_0 \log(r)] = 2\pi\xi \int_0^\infty \mathrm{d}\rho \, 2a_0^2 \rho \,, \\ &= 2\pi\xi \int_0^{1/a_0} \mathrm{d}\rho \, 2a_0^2 \rho + 2\pi\xi \int_{1/a_0}^{\infty} \mathrm{d}\rho \, 2a_0^2 \rho \,, \\ &\to 2\pi\xi \,, \qquad \text{when } a_0 \to 0 \,, \end{aligned} \tag{2.90}$$

since the second integral vanishes in the $a_0 \to 0$ limit.[10] We may now calculate $E_{-|\xi|}$ by applying the inversion symmetry (2.17), and find that $E_{-|\xi|} = 2\pi|\xi|$, so that in general we have

$$E_\xi = 2\pi|\xi| \,. \tag{2.91}$$

[9] For codimensions other than 2, there is an additional condition that relates $A'(r)$ and $\Phi'(r)$. In codimension-2, however, this condition becomes trivial for $A'(r) = 0$.

[10] The evaluation of the integral (2.90) in the limit $a_0 \to 0$ is rather delicate: if the limit is taken prior to integrating, the result would seem to vanish. Yet, the integral itself diverges and so does the tension for any finite a_0. Therefore, the tension becomes an expression of the form $0{\cdot}\infty$ in the $a_0 \to 0$ limit, and is precisely evaluated by means of introducing the intermediate $1/a_0$ limit in the second line of eqs. (2.90).

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Thus, in this limit, the bound (2.88) is satisfied in the modified version:

$$E_\xi \geq \Delta_\xi \,. \tag{2.92}$$

We note that the tension density, i.e., the integrand in eqs. (2.90), is supported either at $r = 0$ (for $a_0 \to 0+$) or at $r = \infty$ (for $a_0 \to 0-$). Hence, the brane solution is now localized, either at the origin or at infinity of the transverse space, as is familiar from the supersymmetric D7-brane solution [23].

2.6.2 Possible F-theory interpretation

We now explore a possible F-theory interpretation to this limit of our solution along the lines of Sen's construction of constant-τ solutions in F-theory [41]. These solutions are classified in terms of the so called j-function,

$$j(\tau(z)) = \frac{f^3}{\delta(f,g)} \,, \tag{2.93}$$

where $f, g, \delta = 4f^3 + 27g^2$ specify the elliptic fiber,

$$u^2 = v^3 + f(z)v + g(z) \tag{2.94}$$

as a function of the location z in the plane perpendicular to the brane. Here $f(z)$ and $g(z)$ are homogeneous functions of degree 8 and 12 respectively. If j is constant, so is then τ — and this is precisely the case in the $a_0 \to 0$ limit of our solutions.

To be more explicit, let us consider the source-free ($\xi = 0$) case in the $a_0 \to 0$. From our previous discussion this can be interpreted as a collection of 12 D7-branes located at the origin, $r = 0$. However, as τ is constant the D7-branes have to be configured such that the total RR-charge is zero.

In order to reproduce this result from F-theory we choose

$$f(z) = \alpha \prod_{i=1,2} (z - z_i)^2 \tilde{f}_4(z) \,, \qquad g(z) = \prod_{i=1,2} (z - z_i)^3 \tilde{g}_6(z) \,, \tag{2.95}$$

which leads to $j(z) \sim \alpha^3$. At each of $z = z_i$ there is a D_4 singularity, whose monodromy is $-\mathbb{1}$. By letting the two singularities coalesce, the total monodromy is $\mathbb{1}$. Thus, we have found agreement with our solution.

Here, the modulus τ is constant because of the fact that the negative RR charge of the orientifold plane is canceled by the D7 branes sitting at the location of the orientifold planes. In general, however, this argument fails, as orientifold interpretations exist only for certain values of τ [41, 42]. For a fixed but arbitrary τ, there does not exist any known orientifold interpretation.

Let us now turn to the $\xi \neq 0$ case. The analysis in section 2.6.1 shows that for $\xi < 0$ (and so also $a_0 \leq 0$) the branes are localized at $r = \infty$. This cluster of branes may then be analyzied separately from the 'background' cluster at $r = 0$. In the supersymmetric limit, $a_0 \to 0$, varying $\xi \in [0, -1]$ corresponds to adding $12|\xi|$ branes at $r = \infty$. Such configurations may be analyzed much the same as was done above for the 'background' cluster, at $r = 0$. In contrast, $\xi > 0$ (and so also $a_0 \geq 0$) the branes are localized at $r = 0$. This cluster of branes then overlaps with the 'background' cluster, and increases the degree of the singularity at $r = 0$. To the best of our knowledge, such highly degenerate configurations have not been discussed in the literature.

Finally we briefly discuss the other solution, $\tau_{\rm II}$. Our analysis of the harmonic function $Z_7(r)$ when $a_0 \to 0$ goes through as before. However, there is a drastic difference compared to τ_I in the supersymmetric limit,

$$\tau_{\rm II} \to -\frac{n}{2\pi} + i\infty\,. \tag{2.96}$$

This is indeed the correct behavior for a collection of n D7-branes located at $r = \infty$ as discussed in the context of F-theory [23, 43].

3. Probing the solution

In this section we continue to study spacetime properties of our background. In the supersymmetric case the background geometry of N D7-branes can be nicely understood by sending a test probe (for example, another D7-brane) from infinity. The physics of the brane probe is described in terms of its effective mass and the effective potential the test brane "feels" in the background it probes [36].

Our background is non-supersymmetric in general. Yet we have shown that in a particular limit we get a supersymmetric background of a certain collection of D7-branes. It is natural thus to probe our solution with a test D7-brane. However, we should keep in mind that although one in principle can probe any background, whether it is made of Dp-branes or not, the resulting physics will in general depend on the type of brane probe. In particular, if a background is a near horizon limit of some set of Dp-branes the most natural probe is a Dp-brane. If a Dp'-brane is used instead this result will have to be consistent with that of the Dp-brane probe analysis [44].

The dynamics of an ideal D7-brane probe[11] is governed by the standard Born-Infeld action [45, 46]:

$$\begin{aligned} S_7 &= \int \mathrm{d}^8x\, \mathcal{L}_7[G^s_{ab}, \Phi, C_8]\,, \\ \mathcal{L}_7 &= -\mu_7 e^{-\Phi}\sqrt{-\det G^s_{ab}} + \mu_7 C_8\,. \end{aligned} \tag{3.1}$$

[11] We emphasize that the brane probes we discuss are ideal in the sense that their presence is assumed not to affect the geometry of the spacetime, and have no back-reaction with the gravitational field probed.

Here G^s_{ab} is the metric on the brane induced from the background string frame metric by embedding the brane coordinates along the spacetime ones. C_8 is the 8-form potential whose field strength is dual to $F_1 = dC_0$, where C_0 is the axion. We use the general notation [36]

$$\mu_p \stackrel{\text{def}}{=} 2\pi(2\pi\sqrt{\alpha'})^{-(p+1)} \tag{3.2}$$

for the brane tensions. All our configurations have $B_{\mu\nu} = 0$, which simplifies the Born-Infeld action [45]. For notational simplicity, we focus on the 'outside' branch, where $r \in [r_s^+, \infty)$. The analysis for the 'inside' branch, where $r \in (0, r_s^-]$, is analogous.

3.1 The $T_{\mu\nu} = 0$ case

Since τ = const., so are the C_0 and C_8 potentials. Similarly, the dilaton is also constant, $e^\Phi = g_s$. The induced string frame metric on the brane probe is [36]

$$[G^s_{ab}] = e^{\Phi/2} g_s^{-1/2} \,\mathrm{diag}[\,(e^{2B}v^2 - e^{2A})\,,\, \underbrace{e^{2A}, \ldots, e^{2A}}_{7}\,]\,, \tag{3.3}$$

where the factor of $e^{2\Phi}$ comes from the relation between the Einstein and string frame metrics, $G^s_{ab} = e^{\Phi/2} G^E_{ab}$, and the factor of $g_s^{-1/2}$ comes from the rescaling of the original action (2.1). The square root of the determinant of this metric is

$$\sqrt{-\det[G^s_{ab}]} = e^{2\Phi} g_s^{-2} Z_7(r) \sqrt{1 - l^2\, Z_7(r)^{-9/8} r^{-2} v^2}\,, \tag{3.4}$$

$$= e^{2\Phi} g_s^{-2} Z_7(r) \left(1 - \frac{1}{2} l^2\, Z_7(r)^{-9/8} r^{-2} v^2 + O(v^4)\right). \tag{3.5}$$

The expansion is in powers of $v^2 = v_r^2 + r^2 v_\theta^2$, the square of the velocity of the brane[12] in the transversal space. As usual, we may assume that the velocity of the brane is small, so that we can neglect terms of $O(v^4)$; we will return to this point shortly.

Inserting eq. (3.5) into (3.1) and truncating at $O(v^2)$, we get the following effective lagrangian density for the test probe,

$$\mathcal{L}_7 \approx \mu_7 [C_8 - g_s^{-1} Z_7(r)] + \frac{1}{2} \mu_7 g_s^{-1} l^2 r^{-2} Z_7(r)^{-1/8} v^2\,. \tag{3.6}$$

Upon integration over unit 7D-volume along the brane, this is simply a non-relativistic lagrangian for a particle with potential and kinetic energies given by

$$V(r) \;=\; \mu_7 V_{D7} [g_s^{-1} Z_7(r) - C_8]\,, \tag{3.7}$$

$$T(r) \;=\; \frac{1}{2} m(r) v^2\,, \tag{3.8}$$

$$m(r) \stackrel{\text{def}}{=} \mu_7 V_{D7} g_s^{-1} l^2 Z_7(r)^{-1/8} r^{-2}\,, \tag{3.9}$$

[12] By working in so called static gauge, the brane coordinates can be aligned with the background. The induced metric carries a time dependence reflecting how the brane moves in the transverse direction.

where $V_{D7} \stackrel{\text{def}}{=} \int \mathrm{d}^7x$ is the spatial 7D-volume of the brane probe. Although the effective mass (3.9) of the brane probe varies with r, it remains positive over the physically permissible region, $r \in [r_s^+, \infty)$ for $a_0 > 0$ and $r \in (\infty, r_s^-]$ for $a_0 < 0$. However, at the naked singularity $m(r)$ diverges. Also unlike in the supersymmetric case, neither the effective potential for the brane probe nor its velocity are constant in the present configurations.

It behooves us then to use the exact expression (3.4), rather than its truncation (3.5). Then, the exact lagrangian density (3.1) becomes:

$$\mathcal{L}_7 = \mu_7 \left[C_8 - g_s^{-1} Z_7(r) \sqrt{1 - l^2\, Z_7(r)^{-9/8} r^{-2} v^2} \right], \tag{3.10}$$

from which we define, performing the Legendre transform, the momentum (assuming $\dot{\theta} = 0$, for simplicity) and the total energy (density):

$$\wp = \frac{\partial \mathcal{L}_7}{\partial v} = \frac{\mu_7 g_s^{-1} l^2 r^{-2} Z_7^{-1/8}}{\sqrt{1 - l^2\, Z_7(r)^{-9/8} r^{-2} v^2}}\, v\,, \tag{3.11}$$

$$\mathcal{E} = \wp\, v - \mathcal{L}_7 = \frac{\mu_7 g_s^{-1} Z_7}{\sqrt{1 - l^2\, Z_7(r)^{-9/8} r^{-2} v^2}} - \mu_7 C_8\,. \tag{3.12}$$

For future reference, we define the 'relativistic' counterparts of (3.7) and (3.8):

$$\mathcal{V}_7 = \mathcal{E}|_{v=0} = \mu_7 g_s^{-1} Z_7 - \mu_7 C_8\,, \tag{3.13}$$

$$\mathcal{T}_7 = \mathcal{E} - \mathcal{V}_7 = \mu_7 g_s^{-1} Z_7 \left[\frac{1}{\sqrt{1 - l^2\, Z_7(r)^{-9/8} r^{-2} v^2}} - 1 \right], \tag{3.14}$$

From the form of the total energy (density), we may interpret

$$c_{\text{eff}} = r l^{-1}\, Z_7(r)^{9/16} \tag{3.15}$$

as the effective speed of light for the brane probe approaching the naked singularity, and note that $\lim_{r \to r_s^+} c_{\text{eff}} = 0$.

Next, we assume that the total energy (density), $\mathcal{E}$, is conserved[13] and may be used to parametrize the velocity:

$$v = \frac{r}{l} Z_7(r)^{9/16} \sqrt{1 - \left(\frac{\mu_7 g_s^{-1} Z_7(r)}{\mathcal{E} + \mu_7 C_8} \right)^2}\,. \tag{3.16}$$

For the velocity of the brane probe to be real (as appropriate for the classical analysis), it must be that:

$$Z_7(r) \geq 0 \qquad \text{and} \qquad 1 \geq \left(\frac{\mu_7 g_s^{-1} Z_7(r)}{\mathcal{E} + \mu_7 C_8} \right)^2. \tag{3.17}$$

[13]The conservation of total energy is reasonable, as the brane probe is assumed to be ideal: it probes the spacetime geometry and so interacts only with the effective potential. Thus, unlike a realistic probe, it cannot lose energy through radiation or decay.

The first of these implies that $r \geq r_{\rm s}^{+} = e^{-1/a_0}$, as was established in section 2.4. Taking the square-root of both sides of the second condition (3.17), we obtain the lower bound on the total energy (density):

$$\mathcal{E} + \mu_7 C_8 \geq \mu_7 g_s^{-1} Z_7(r)\,, \qquad \text{where } Z_7(r) \geq 0\,. \tag{3.18}$$

Solving this for r, we find the outer limit:

$$r \leq r_{\rm o} \stackrel{\rm def}{=} \exp\Big\{a_0^{-1}\Big(\frac{\mathcal{E} + \mu_7 C_8}{g_s^{-1}\mu_7} - 1\Big)\Big\}\,. \tag{3.19}$$

From eq. (3.16) (see also the plot of $v(r)$ in figure 5), as the brane probe approaches $r_{\rm o}$ from 'inside' or $r_{\rm s}^{+}$ from 'outside,' its velocity drops to zero, and the brane probe starts returning within the 'window' $r \in [r_{\rm s}^{+}, r_{\rm o}]$. (By conservation of energy, the brane probe will continue moving after hitting the naked singularity.) Both $r_{\rm s}^{+}$ and $r_{\rm o}$ are classical turning points, and the brane probe is being ricocheted between them. This result is in perfect accord with the result in [1, appendix A], from which it follows that a quantum particle probe must remain in a scattered state although the potential is attractive. Note though that the transverse volume available to the brane probe is determined by size of the 'window' $r \in [r_{\rm s}^{+}, r_{\rm o}]$; see section 4.3, rather than $r \in [r_{\rm s}^{+}, \infty]$.

One of the assumptions of the brane probe analysis is that the branes are moving *very slowly* in the given background. We are now in the position to make this statement more precise for the present configuration. By substituting the exact result (3.16) into the lagrangian density (3.10), it is easy to verify that the square-root (3.4), and so also the entire Born-Infeld action are real. Note that this is equivalently ensures that the 'lapse function' along the brane probe, $G_{tt} = (e^{2B} - e^{2A})$ in the induced metric (3.3), remains negative. Thus, the spacetime of the brane probe remains of the lorentzian signature everywhere in the physically permitted transverse space. Moreover, the Taylor expansion of the square-root (3.4) always converges, owing to (3.16), and also the $O(v^4)$ terms are *always* smaller than the $O(v^2)$ ones:

$$\frac{1}{2} l^2\, Z_7(r)^{-9/8} r^{-2} v^2 > \frac{1}{8}(l^2\, Z_7(r)^{-9/8} r^{-2} v^2)^2\,. \tag{3.20}$$

This is, owing to the exact result (3.16), equivalent to

$$3 > -\Big(\frac{\mu_7 g_s^{-1} Z_7(r)}{\mathcal{E} + \mu_7 C_8}\Big)^2\,, \tag{3.21}$$

which is trivially satisfied and restricts neither r nor $\mathcal{E}$. Hence the description of a classical particle moving in a potential $V(r)$ and with kinetic energy $T(r)$ given by (3.7)–(3.9) provides a consistent approximation when $r \in [r_{\rm s}^{+}, \infty]$.

Note however that as $r \to r_{\rm s}^{+}$, both the velocity of the probe (3.16) and the effective speed of light (3.15) go to zero, in such a way that

$$\left(\frac{v}{c_{\rm eff}}\right) = \sqrt{1 - \left(\frac{\mu_7 g_s^{-1} Z_7(r)}{\mathcal{E} + \mu_7 C_8}\right)^2} \to 1\,. \tag{3.22}$$

Furthermore, near $r_{\rm s}^{+}$, all of the brane probe's energy becomes 'kinetic'. This suggests that neglecting the relativistic effects such as gravitational back-reaction, perturbation of the background, etc. is no longer warranted. To ensure that the 'non-relativistic' formulae (3.7)–(3.9) are still reliable, we require that the difference between the relativistic kinetic energy (3.14) and the non-relativistic one (3.8) be smaller than the latter:

$$\mathcal{T}_7(r) - T(r)V_{D7}^{-1} < T(r)V_{D7}^{-1}\,, \quad \text{i.e.,} \quad \mathcal{T}_7(r) < 2T(r)V_{D7}^{-1}\,. \tag{3.23}$$

Using the exact result for the velocity (3.16), this produces the *upper* bound on the total energy (density):

$$\mathcal{E} + \mu_7 C_8 \leq \beta\, \mu_7 g_s^{-1} Z_7(r)\,, \qquad \beta \stackrel{\rm def}{=} \sqrt{\frac{2}{3-\sqrt{5}}} \approx 1.618\,. \tag{3.24}$$

In turn, this produces an *inner* limit on the radius:

$$r \geq r_{\rm i} \stackrel{\rm def}{=} \exp\left\{a_0^{-1}\left(\frac{\mathcal{E} + \mu_7 C_8}{\beta g_s^{-1} \mu_7} - 1\right)\right\}\,. \tag{3.25}$$

Now, we may keep reducing[14] the total energy (density) of the brane probe towards the lower limit (3.18), $\mathcal{E} \to -\mu_7 C_8$, in the hopes of decreasing $r_{\rm i} \to r_{\rm s}^{+}$ so as to approach the naked singularity. This however also reduces $r_{\rm o} \to r_{\rm s}^{+}$, and the allowed region, $[r_{\rm i}, r_{\rm o}]$, for the brane probe becomes vanishingly small as it nears the naked singularity: the (classical) brane probe is trapped by it. Alternatively, $r_{\rm i} \to r_{\rm s}^{+}$ by taking $a_0 \to 0$ while keeping $\frac{\mathcal{E}+\mu_7 C_8}{\mu_7 g_s^{-1}} > 1$, as can be seen from eqs. (3.19) and (3.25).

The two limits, (3.19) and (3.25), provide a region in which the classical probe analysis based on the action (3.1) is limited:

$$r_{\rm s}^{+} < r_{\rm i} \leq r \leq r_{\rm o}\,, \tag{3.26}$$

where the left-most inequality holds since $\mathcal{E} + \mu_7 C_8 \geq 0$, owing to the 'outer reality' condition (3.18). As discussed above, the outer limit $r_{\rm o}$ is a simple classical turning

[14] The brane probe here is an ideal object, the total energy density of which is simply a control; we do not address the nature of the physical mechanism by which to decrease the brane probe's energy. However, it is conceivable that a real probe would lose energy through gravitational bremsschtrahlung, if nothing else.

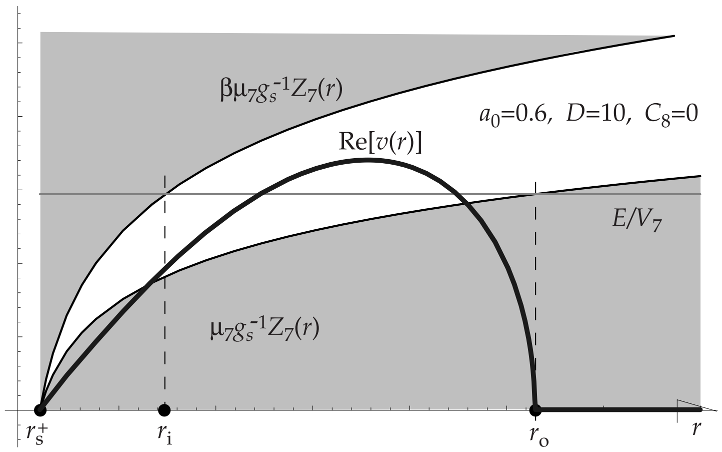

Figure 5: The 'outer' limit curve (lower) and the 'inner' limit curve (upper); shaded areas are forbidden. The brane probe velocity, $v(r)$ as defined in eq. (3.16), is plotted (grey). Although v decreases as $r < r_{\rm i}$, $v/c_{\rm eff} \approx 0.786$ at $r_{\rm i}$ and *grows* as $r \to r_{\rm s}^{+}$. Within $[r_{\rm s}^{+}, r_{\rm i}]$, the bound (3.23) is violated; the brane probe based on the action (3.1) is no longer reliable.

point, while below $r_{\rm i}$ the action (3.1) no longer suffices to describe the dynamics of the brane probe. The distance from the naked singularity to these limits grows exponentially, but remains finite for all finite energies. By comparison, there is no 'outer' turning point for higher codimension brane probes [36, 47]: there the velocity is constant and an outgoing brane probe would leave the singularity forever. Obviously, this distinction happens because, unlike its higher codimension analogues, $Z_7(r)$ has no asymptotic region and in fact diverges at $r = \infty$.

The situation is illustrated in figure 5.

The radii where a given energy density level crosses the two limiting curves are $r_{\rm o}$ and $r_{\rm i}$, respectively. In turn, eqs. (3.18) and (3.24) can be combined to produce the extended inequality

$$\mu_7[g_s^{-1}Z_7(r) - C_8] \leq \mathcal{E} \leq \mu_7[\beta g_s^{-1}Z_7(r) - C_8]\,. \tag{3.27}$$

Here both the upper and the lower limit depend on r, through $Z_7(r)$. (While there does exist an analogous 'band' of allowed brane probe energies in the higher codimension case, its lower limit is constant [47].) It is of course possible to choose the constant dual-axion field strength so that the lower limit on the total energy of the brane probe would vanish at any particular point of interest: $C_8 = g_s^{-1}$ makes it vanish in the surrogate asymptotic region $r \sim 1$. Furthermore, the width of the 'band' of energies allowed by the inequalities (3.27) is

$$\Delta\mathcal{E} = (\beta-1)\mu_7 g_s^{-1}Z_7(r) = 2(\beta-1)\pi(2\pi\sqrt{\alpha'})^{-8}g_s^{-1}Z_7(r)\,, \tag{3.28}$$

and varies with $Z_7(r)$.

In the limit $a_0 \to 0$, the inequalities (3.27) simplify to

$$\mu_7[g_s^{-1} - C_8] \le \mathcal{E} \le \mu_7[\beta g_s^{-1} - C_8]\,. \tag{3.29}$$

Again, we may choose the constant dual-axion field strength, $C_8 \to g_s^{-1}$, whereupon the total energy of the brane probe is limited to be $0 \le \mathcal{E} \le (\beta-1)\mu_7 g_s^{-1}$. With the total energy within these limits, we now find that

$$\lim_{a_0\to 0} r_{\rm o} = +\infty\,, \qquad \text{and} \qquad \lim_{a_0\to 0} r_{\rm i} = 0 = \lim_{a_0\to 0} r_{\rm s}^{+}\,. \tag{3.30}$$

That is, in the $a_0 \to 0$ limit, the probe analysis based on the expansion (3.5) is valid in the whole $r \in [0,\infty)$ region.

Next, $V(r)$ becomes a constant and the mass (3.9) of the probe moving around the cylinder is constant. In addition, the mass of the probe moving in the radial direction, when measured in the proper distance, is also constant. This is consistent with supersymmetry. It is hence satisfying to see how the probe analysis reproduces this fact.

Finally, even the perhaps somewhat curious looking region of allowed energies (3.29) can also be derived [47] from the standard analysis of supersymmetric 7-branes [36].

3.2 The $T_{\mu\nu} \neq 0$ case

3.2.1 τ_I

This case resembles the source-free situation in that $\mathrm{Re}\, e(\tau_I) = $ const. and hence both $C_{0,8}$ are constant although the dilaton is now non-trivial

$$e^{2\Phi(\theta)} = g_s^2 e^{-2\omega\theta}\,. \tag{3.31}$$

The probe analysis follows along the lines of section 3.1. From eq. (3.1) and the expansion of the determinant of the induced metric, we find that the lagrangian is given by

$$\mathcal{L}_7 = \mu_7[C_8 - g_s^{-1}e^{-\omega\theta}Z_7(r)] + \frac{1}{2}\mu_7 g_s^{-1} l^2 e^{-\omega\theta} r^{-2} Z_7(r)^{-1/8} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_7^2(r)} v^2 + O(v^4)\,. \tag{3.32}$$

Upon integration over unit 7D-volume along the brane, this again is a lagrangian for a particle moving with potential and kinetic energies

$$\begin{aligned} V(r,\theta) &= \mu_7 V_{D7}[g_s^{-1}e^{-\omega\theta}Z_7(r) - C_8] \\ T(r,\theta) &= \frac{1}{2}m(r,\theta)v^2\,, \\ m(r,\theta) &\stackrel{\rm def}{=} \mu_7 V_{D7} g_s^{-1} l^2 e^{-\omega\theta} Z_7(r)^{-1/8} r^{-2} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_7^2(r)}\,. \end{aligned} \tag{3.33}$$

In analogy with our previous discussion, when $T_{\mu\nu} = 0$, the 'outer' and 'inner' reality conditions are

$$\mathcal{E} + \mu_7 C_8 \geq \mu_7 g_s^{-1} e^{-\omega\theta} Z_7(r)\,, \quad \text{where} \quad Z_7(r) \geq 0\,, \tag{3.34}$$

$$\text{i.e.} \quad r \leq r_{\rm o} \stackrel{\rm def}{=} \exp\left\{a_0^{-1}\left(\frac{\mathcal{E} + \mu_7 C_8}{g_s^{-1} e^{-\omega\theta}\mu_7} - 1\right)\right\}, \tag{3.35}$$

and

$$\beta\, \mu_7 g_s^{-1} e^{-\omega\theta} Z_7(r) \geq \mathcal{E} + \mu_7 C_8\,, \tag{3.36}$$

$$\text{i.e.} \quad r \geq r_{\rm i} \stackrel{\rm def}{=} \exp\left\{a_0^{-1}\left(\frac{\mathcal{E} + \mu_7 C_8}{\beta g_s^{-1} e^{-\omega\theta}\mu_7} - 1\right)\right\}, \tag{3.37}$$

respectively. As for the $T_{\mu\nu} = 0$ case, from (3.35) and (3.37), we get a window of allowed values for r,

$$r_{\rm s}^{+} < r_{\rm i} \leq r \leq r_{\rm o}\,. \tag{3.38}$$

Note that the classical limits $r_{\rm i}$ and $r_{\rm o}$ now depend on θ. eqs. (3.34) and (3.36) then imply an extended inequality analogous to (3.27),

$$\mu_7[g_s^{-1} e^{-\omega\theta} Z_7(r) - C_8] \leq \mathcal{E} \leq \mu_7[\beta g_s^{-1} e^{-\omega\theta} Z_7(r) - C_8]\,. \tag{3.39}$$

Here both the upper and the lower limit depend on r, through $Z_7(r)$, and θ through the dilaton dependence, $e^{\Phi(\theta)}$. Owing to the θ-depenence of the limits, the limits on the brane probe total energy can be simplified only in a fixed, albeit arbitrary direction θ_a. Furthermore, the width of the 'band' of energies allowed by the inequalities (3.39) is

$$\Delta\mathcal{E} = (\beta - 1)\mu_7 g_s^{-1} e^{-\omega\theta} Z_7(r) = 2(\beta - 1)\pi\left(2\pi\sqrt{\alpha'}\right)^{-8} g_s^{-1} e^{-\omega\theta} Z_7(r)\,, \tag{3.40}$$

and varies with $Z_7(r)$ and $e^{\omega\theta}$.

In the limit $a_0 \to 0$, the inequalities (3.39) simplify to

$$\mu_7[g_s^{-1} - C_8] \leq \mathcal{E} \leq \mu_7[\beta g_s^{-1} - C_8]\,. \tag{3.41}$$

We may choose $C_8 \to g_s^{-1}$, so the total energy of the brane probe is limited to be

$$0 \leq \mathcal{E} \leq (\beta - 1)\mu_7 g_s^{-1} \quad \text{for} \quad \theta = \theta_a\,. \tag{3.42}$$

With the total energy within these limits, we now find that

$$\lim_{a_0\to 0} r_{\rm o} = +\infty\,, \qquad \text{and} \qquad \lim_{a_0\to 0} r_{\rm i} = 0 = \lim_{a_0\to 0} r_{\rm s}^{+}\,. \tag{3.43}$$

That is, in the $a_0 \to 0$ limit, the probe analysis based on the expansion (3.5) is valid in the whole $r \in [0, \infty)$ region.

As before $V(r)$ and the mass of the probe moving around the cylinder become constant. Also, the mass of the probe moving in the radial direction, when measured in the proper distance, is constant. Both of these facts are again consistent with supersymmetry.

3.2.2 $\tau_{\rm II}$

From the expression for $\tau_{\rm II}$ we have

$$\begin{aligned} C_8(r) &= \frac{\omega}{a_0}\tanh(\omega\pi) Z_7^2(r) C_8^{(0)}\,, \\ e^{\Phi(\theta)} &= 2g_s \tanh(\omega\pi)\cosh(\omega\theta)\,. \end{aligned} \tag{3.44}$$

The probe analysis follows along the lines of section 3.1. From eq. (3.1) and the expansion of the determinant of the induced metric, we find that the lagrangian is given by

$$\mathcal{L}_7 = \mu_7[C_8(r) - g_s^{-2}e^{\Phi(\theta)}Z_7(r)] + \frac{1}{2}\mu_7 l^2 g_s^{-2} e^{\Phi(\theta)} Z_7(r)^{-1/8} r^{-2-\xi(2+a_0\log(r))} v^2 + O(v^4)\,. \tag{3.45}$$

Upon integration over unit 7D-volume along the brane, the effective potential and kinetic energies of the probe are given by

$$\begin{aligned} V(r,\theta) &= \mu_7 V_{D7}[g_s^{-2}e^{\Phi(\theta)}Z_7(r) - C_8(r)] \\ T(r,\theta) &= \frac{1}{2}m(r,\theta)v^2\,, \\ m(r,\theta) &\stackrel{\rm def}{=} \mu_7 V_{D7} l^2 g_s^{-2} e^{\Phi(\theta)} Z_7(r)^{-1/8} r^{-2-\xi(2+a_0\log(r))}\,. \end{aligned} \tag{3.46}$$

As discussed in section 3.1 assuming that the total energy, E, is conserved, we find the following range of allowed energies

$$\mu_7[g_s^{-2}e^{\Phi(\theta)}Z_7(r) - C_8(r)] \le \mathcal{E} \le \mu_7[\beta g_s^{-2}e^{\Phi(\theta)}Z_7(r) - C_8(r)]\,. \tag{3.47}$$

Here both the upper and the lower limit depend on r, through $Z_7(r)$ and $C_8(r)$, and θ through $e^{\Phi(\theta)}$. As before, (3.47) determines the 'inner' and 'outer' limits, $r_{\rm i}$ and $r_{\rm o}$.

In contrast to the analysis in sections 3.1 and 3.2 in the limit $a_0 \to 0$, the inequalities (3.47) enforce $\mathcal{E} = -\mu_7 C_8$ rather than a window of allowed energies. The solutions for $r_{\rm i}$ and $r_{\rm o}$ can be obtained in analogy with eq. (3.30),

$$\lim_{a_0\to 0} r_{\rm o} = \infty\,, \qquad \text{and} \qquad \lim_{a_0\to 0} r_{\rm i} = \infty = \lim_{a_0\to 0} r_{\rm s}^+\,. \tag{3.48}$$

Thus, in the $a_0 \to 0$ limit, the probe analysis breaks down.

4. Non-supersymmetric resolution of naked singularities

We found in the last section that useful information can be learned by studying a non-supersymmetric background with a supersymmetric Dp brane. While this particular set-up is interesting in its own right, we now turn to a potentially more exciting configuration in which the background is wrapped on a compact four-dimensional manifold, such as K3. From a phenomenological point of view we are eventually interested in a four-dimensional theory. On the other hand, K3's non-trivial curvature induces an effective negative number of D3-branes. This leads to a situation which is analogous to the enhançon-mechanism [33], although in a non-supersymmetric setting. While we will compute the effect of the wrapping on the exponential hierarchy, we will focus on the formal aspects of this problem.

4.1 The $T_{\mu\nu} = 0$ case

We start by looking at the solution compactified on a T^4. By T-duality, the 7-brane is turned into a 3-brane, which however is smeared along the T^4. The metric, dilaton and four-form potential are obtained, by following the analysis of ref. [36]:

$$\begin{aligned} ds^2 &= Z_3^{2/4}\eta_{ab}dx^a dx^b + V_{T^4}^{1/2} ds^2_{T^4} + l^2 Z_3^{-5/8} r^{-2}(dr^2 + r^2 d\theta^2)\,, \\ e^{2\Phi} &= g_s^2 Z_3^{-1}\,, \\ C_4 &= \text{const.}\,, \\ Z_3 &= 1 + a_0 \frac{(2\pi\alpha'^{1/2})^4}{V_{T^4}} \log(r)\,. \end{aligned} \tag{4.1}$$

Note that the non-trivial dilaton compensates for the $Z_3^{-5/8}$ behavior rather than $Z_3^{-3/4}$ as would have been expected for a 3-brane in $D = 6$ from our solution (2.8)-(2.9). In fact, the behavior of the dilaton is opposite to that of the supersymmetric D3- and D7-branes. There the D3-brane has a constant dilaton while the D7-brane has a logarithmic running.

In wrapping a supergravity background on a K3 there is a non-trivial $R \wedge R$ coupling, absent for the T^4. This has the effect of inducing a negative number of 3-branes [48]. We will assume that this is the only non-trivial coupling between the K3 and background although there is no supersymmetry to protect other couplings from contributing[15] Therefore we obtain the following wrapped solution

$$ds^2 = Z_3^{2/4} Z_7^{1/4}\eta_{ab}dx^a dx^b + V_{K3}^{1/2} Z_7^{1/4} ds^2_{K3} + l^2 Z_3^{-5/8} Z_7^{-7/8} r^{-2}(dr^2 + r^2 d\theta^2)\,, \tag{4.2}$$

$$e^{2\Phi} = g_s^2 Z_3^{-1}\,, \tag{4.3}$$

$$C_4 = \text{const.}, \tag{4.4}$$

$$C_8 = \text{const.}\,, \tag{4.5}$$

$$Z_3 = 1 - a_0 \frac{(2\pi\alpha'^{1/2})^4}{V_{K3}} \log(r)\,, \tag{4.6}$$

$$Z_7 = 1 + a_0 \log(r)\,, \tag{4.7}$$

where we have used the standard procedure for generating a supergravity solution of intersecting branes [40]. We however wish to comment on the form of $Z_3(r)$. Firstly,

[15] This assumption is justified by the self-consistency of the analysis conducted below; in the $a_0 \to 0$ limit we recover the results of [33].

the $\log(r)$ behavior is due the the 3-brane being smeared over the 4-dimensional K3, so that the true transverse space is still 2-dimensional. In addition, we know that $Z_3(r) \to 1$ if $V_{K3} \to \infty$, i.e., if the volume of K3 becomes infinite and we have effectively uncompactified the spacetime. This rescales the a_0 parameter, the precise form of which is determined by dimensional analysis as shown in (4.6). Finally the relative sign in $Z_3(r)$ is determined along the lines of the argument, given in ref. [48], for the emergence of a negative number of 3-branes.

With the wrapped solution from (4.7) at hand we turn to the probe analysis. Because of the $R \wedge R$ coupling the probe action has to be modified (see e.g. [36])

$$S = -\int_M e^{-\Phi}(\mu_7 V_{K3}(r) - \mu_3)\sqrt{-\det G^s_{ab}} + \mu_7 \int_{M\times K3} C_8 - \mu_3 \int_M C_4 \,. \tag{4.8}$$

Here,

$$V_{K3}(r) = V_{K3}\left(\frac{Z_7}{Z_3}\right)^{1/2}, \tag{4.9}$$

is the volume of the K3 in the string frame metric. As in the unwrapped case $C_{4,8}$ are both constant.

In order to understand the important features of this expression let us briefly recapitulate the salient features in the supersymmetric case; for more details, see [33]. First, for a set of supersymmetric D7-branes wrapped on a K3, the potential seen by a D7-brane probe is constant. Second, the kinetic term has a mass $m_{3,7} \sim (\mu_7 V_{K3} Z_3 - \mu_3 Z_7)$. However, while the harmonic function for D7-brane is the same as in our case, the interpretation of a_0 is as the RR-charge. Because of the special properties of solutions in codimension 2, there is no asymptotically flat region, except for the 'surrogate asymptotic region' around $r = 1$. This means in particular that the standard supersymmetric 7-branes with RR-charge $a_0 > 0$ is actually located at $r = \infty$. When we wrap the 7-branes on a K3, we have in fact naked singularities at each of the zero locus for Z_7 and Z_3 respectively, where $Z_3 = 0$ is the location of the repulson [33]. The enhançon radius occurs at a radius where the effective tension, $\mu^s_{3,7} \stackrel{\text{def}}{=} (\mu_7 V^s_{K3}(r) - \mu_3)$ vanishes, i.e., where the volume of the K3 reaches the value $\mu_3/\mu_7 = (2\pi\sqrt{\alpha'})^4$

$$V^s(r) \equiv V^s_{K3}\frac{Z_3(r)}{Z_7(r)} = \frac{\mu_3}{\mu_7}\,. \tag{4.10}$$

Thus, the repulson singularity at $Z_3 = 0$ is resolved and the branes which were located at the repulson singularity have now moved *inwards* to the enhançon radius implicitly given by the expression for $V^s(r)$ above.

Returning to (4.8), we note the difference between the supersymmetric expression (4.10) and the corresponding result in our case (4.9). Hence, the action and the tension for our 3-7 brane system, vanishes when

$$\mu_{3,7} \stackrel{\text{def}}{=} \mu_7 V_{K3}\left(\frac{Z_7(r)}{Z_3(r)}\right)^{1/2} - \mu_3 = 0\,. \tag{4.11}$$

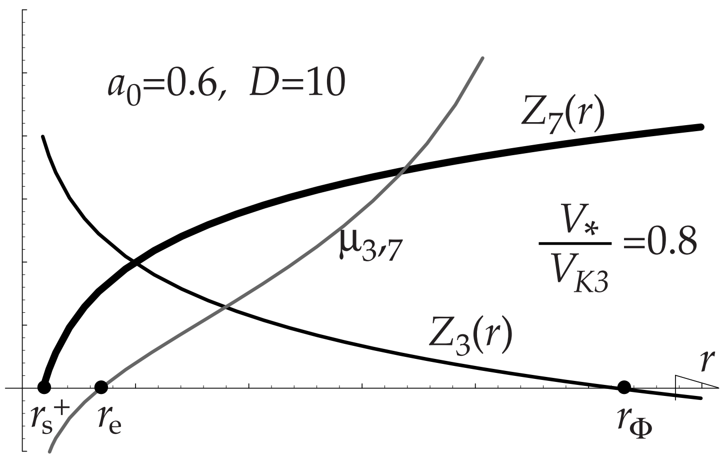

Figure 6: A sketch of the physically permissible region, with the marked locations of r_s^+, r_e and r_Φ.

This happens at the location which we will call the enhançon radius:

$$r_e \stackrel{\text{def}}{=} e^{(V_*/V_{K3}-1)/a_0}\,, \qquad \text{where } V_* \stackrel{\text{def}}{=} (2\pi\alpha'^{1/2})^4\,. \tag{4.12}$$

Note that in the $V_{K3} \to \infty$ limit, or alternatively $\alpha' \to 0$, $r_e \to r_s^+$.

Let us focus on the $a_0 > 0$ case; the $a_0 < 0$ case can then easily be obtained by sending $r \to 1/r$. There appear three marked locations in the physically permissible domain of r:

1. $r_s^+ = e^{-a_0^{-1}}$, where $Z_7(r) = 0$, and which is the lower limit of the physically permissible region;
2. $r_e = e^{(V_*/V_{K3}-1)a_0^{-1}} \geq r_s^+$, where the effective tension, $\mu_{3,7}$ vanishes;
3. $r_\Phi = e^{\frac{V_{K3}}{V_*}a_0^{-1}} \geq r_e$, where $Z_3(r) = 0$, and the dilaton diverges owing to eq. (4.3).

Clearly, near and beyond r_Φ, the stringy effects become too strong for the present analysis which needs to be further refined; our present conclusions are therefore limited to $r < r_\Phi$.

From a naive supergravity point of view, the wrapped 7-branes are located at the naked singularity, r_s^+. However, the brane probe does not detect this singularity. Instead, the wrapped 7-branes have effectively moved out from r_s^+ to r_e. Still, we have to make sure that the resolution of the naked singularity takes place in a region in which the probe analysis is valid. We therefore turn to this task.

Following the analysis in section 3.1 we have a lagrangian for the test probe given by

$$\begin{aligned}\mathcal{L} = & -g_s^{-1} Z_3 Z_7 \left(\mu_7 V_{K3} Z_3^{-1/2} - \mu_3 Z_7^{-1/2}\right) + \mu_7 V_{K3} C_8 - \mu_3 C_4 + \\ & + \frac{1}{2} l^2 g_s^{-1} r^{-2} Z_3^{-1/8} Z_7^{-1/8} \left(\mu_7 V_{K3} Z_3^{-1/2} - \mu_3 Z_7^{-1/2}\right) v^2 + O(v^4)\,.\end{aligned} \tag{4.13}$$

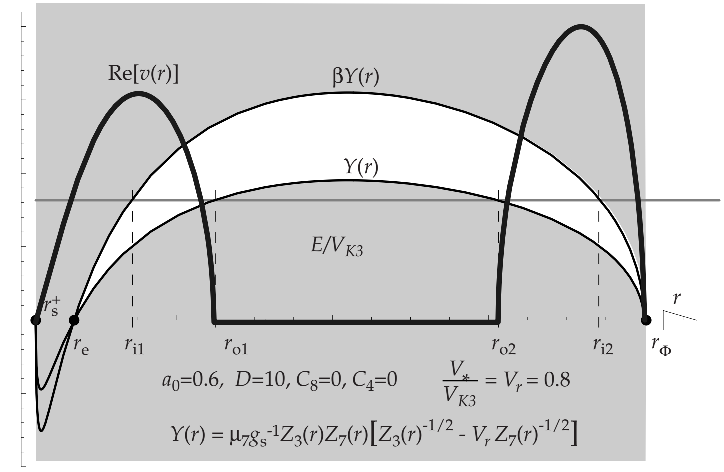

Figure 7: A sketch of the physically permissible region, with the marked locations of r_s^+, r_e, $r_{i1,2}$, $r_{o1,2}$ and r_Φ.

In a supersymmetric setting one finds that the effective potential for the brane probe is constant. However, that is not the case above. As in section 3.1 we assume that the total energy, E, is conserved and find the following condition on the energy density $\mathcal{E}$,

$$\begin{aligned} \mu_7 \left[g_s^{-1} Z_3(r) Z_7(r) \left(Z_3^{-1/2} - \frac{V_*}{V_{K3}} Z_7^{-1/2} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] &\leq \mathcal{E} \,, \\ \mu_7 \left[\beta g_s^{-1} Z_3(r) Z_7(r) \left(Z_3^{-1/2} - \frac{V_*}{V_{K3}} Z_7^{-1/2} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] &\geq \mathcal{E} \,. \end{aligned} \tag{4.14}$$

These inequalities provide a region of allowed energies for the probe to be at a given position, see figure 7. Note that compared to the unwrapped case, it is possible to have two physically permissible regions, $r \in [r_{i1}, r_{o1}]$ and $r \in [r_{o2}, r_{i2}]$. We defer a more detailed analysis of this point to [47].

In the limit $a_0 \to 0$, the inequalities (4.14) simplify to

$$\mu_7 \left[g_s^{-1} \left(1 - \frac{V_*}{V_{K3}} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] \leq \mathcal{E} \leq \mu_7 \left[\beta\, g_s^{-1} \left(1 - \frac{V_*}{V_{K3}} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] . \tag{4.15}$$

From figure 7 we find that the range of validity is given by r_i and r_o:

$$\lim_{a_0 \to 0} r_{o1} = +\infty \,, \qquad \text{and} \qquad \lim_{a_0 \to 0} r_{i1} = 0 = \lim_{a_0 \to 0} r_s^+ \,. \tag{4.16}$$

That is, in the $a_0 \to 0$ limit, the probe analysis based on the expansion (3.5) is valid in the whole $r \in [0, \infty)$ region. Finally, both the potential and kinetic terms go to constants in (4.13) and we are back in a supersymmetric scenario.

4.2 The $T_{\mu\nu} \neq 0$ case

4.2.1 τ_I

This case resembles the source-free situation in that $\mathrm{Re}\, e\tau_I = \text{const.}$ and hence both $C_{0,8}$ are constant. Thus, the solution is given by the following expressions,

$$
\begin{aligned}
ds^2 &= Z_3^{2/4} Z_7^{1/4} \eta_{ab} dx^a dx^b + V_{K3}^{1/2} Z_7^{1/4} ds_{K3}^2 + \\
&\quad + l^2 Z_3^{-5/8} Z_3^{-7/8} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_7^2} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_3^2} r^{-2} (dr^2 + r^2 d\theta^2) && (4.17)\\
e^{2\Phi} &= g_s^2 e^{-2\omega\theta} Z_3^{-1} && (4.18)\\
C_4 &= \text{const.}, && (4.19)\\
C_8 &= \text{const.}, && (4.20)\\
Z_3 &= 1 - a_0 \frac{(2\pi\alpha'^{1/2})^4}{V_{K3}} \log(r) && (4.21)\\
Z_7 &= 1 + a_0 \log(r)\,. && (4.22)
\end{aligned}
$$

Although there is a non-trivial θ dependence in the dilaton which gives a slight modification to the metric the probe analysis goes through as in section 4.1,

$$
\begin{aligned}
\mathcal{L} \approx &-g_s^{-1} e^{-\omega\theta} Z_3^{1/2} Z_7^{1/2} \left(\mu_7 V_{K3} Z_7^{1/2} - \mu_3 Z_3^{1/2} \right) + \mu_7 V_{K3} C_8 - \mu_3 C_4 + \\
&+ \frac{1}{2} l^2 g_s^{-1} e^{\omega\theta} \left(\prod_{i=3,7} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_i^2} Z_i^{-9/8} \right) r^{-2} \left(\mu_7 V_{K3} Z_7^{1/2} - \mu_3 Z_3^{1/2} \right) v^2\,. \qquad (4.23)
\end{aligned}
$$

As in section 4.1 we assume that the total energy, E, is conserved and find the following condition on the energy density $\mathcal{E}$,

$$
\begin{aligned}
\mu_7 \left[g_s^{-1} e^{-\omega\theta} Z_3(r) Z_7(r) \left(Z_3^{-1/2} - \frac{V_*}{V_{K3}} Z_7^{-1/2} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] &\leq \mathcal{E}\,, \\
\mu_7 \left[\beta\, g_s^{-1} e^{-\omega\theta} Z_3(r) Z_7(r) \left(Z_3^{-1/2} - \frac{V_*}{V_{K3}} Z_7^{-1/2} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] &\geq \mathcal{E}\,. \qquad (4.24)
\end{aligned}
$$

These inequalities provide a region of allowed energies for the probe to be at a given position. For any given direction, θ, in the transverse space we can sketch the physical permissible region as in figure7.

In the limit $a_0 \to 0$, the inequalities (4.24) simplify to

$$
\mu_7 \left[g_s^{-1} \left(1 - \frac{V_*}{V_{K3}} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] \leq \mathcal{E} \leq \mu_7 \left[\beta\, g_s^{-1} \left(1 - \frac{V_*}{V_{K3}} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right]. \qquad (4.25)
$$

Following the same procedure as in the previous section, we find the following range of validity:

$$
\lim_{a_0 \to 0} r_{\mathrm{o}} = +\infty\,, \qquad \text{and} \qquad \lim_{a_0 \to 0} r_{\mathrm{i}} = 0 = \lim_{a_0 \to 0} r_{\mathrm{s}}^{+}\,. \qquad (4.26)
$$

That is, in the $a_0 \to 0$ limit, the probe analysis based on the expansion (3.5) is valid in the whole $r \in [0, \infty)$ region.

Finally, in the limit $a_0 \to 0$ the potential term in (4.23) becomes a constant. However, the kinetic energy has an r-dependence from the non-trivial dilaton, $r^{\xi(-1+(2\pi\sqrt{\alpha'})^4/V_{K3})}$, in addition to the cylindrical r^{-2} factor. The exponent reflects the deficit angle created by the wrapped 7-brane. How does this compare to the supersymmetric case? A collection of supersymmetric 7-branes located at the $r = \infty$ has the same $r^{-\xi}$ behavior in the metric to reflect a deficit angle $\Delta = 2\pi|\xi|$, i.e., the transverse space is a cone rather than a plane. By redefining the coordinates on the transverse space, we can thus absorb this conical behavior in the induced metric on the brane probe, and hence in the velocity of the brane probe. This argument can be repeated for the wrapped case. In the end, the effective mass of the probe is constant.

4.2.2 τ_{II}

Based on our earlier arguments the wrapped τ_{II} solution is given by

$$\begin{aligned} ds^2 &= Z_3^{2/4} Z_7^{1/4} \eta_{ab} dx^a dx^b + V_{K3}^{1/2} Z_7^{1/4} ds_{K3}^2 + \\ &\quad + l^2 Z_3^{-5/8} Z_3^{-7/8} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_7^2} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_3^2} r^{-2} (dr^2 + r^2 d\theta^2) \qquad (4.27) \\ e^{2\Phi} &= g_s^2 f(\theta)^2 Z_3^{-1} , \qquad \text{where } f(\theta) \stackrel{\text{def}}{=} 2\tanh(\omega\pi)\cosh(\omega\theta) \qquad (4.28) \\ C_4 &= \frac{\omega}{a_0} \tanh(\omega\pi) Z_3^2(r) \qquad (4.29) \\ C_8 &= \frac{\omega}{a_0} \tanh(\omega\pi) Z_7^2(r) \qquad (4.30) \\ Z_3 &= 1 - a_0 \frac{(2\pi\alpha'^{1/2})^4}{V_{K3}} \log(r) \qquad (4.31) \\ Z_7 &= 1 + a_0 \log(r) . \qquad (4.32) \end{aligned}$$

Since the RR-potentials that couple to the 3 and 7-branes are non-trivial the effective lagrangian as seen by the the brane probe is given by (to $O(v^2)$)

$$\begin{aligned} \mathcal{L} = &-g_s^{-1} f(\theta) Z_3^{1/2} Z_7^{1/2} \left(\mu_7 V_{K3} Z_7^{1/2} - \mu_3 Z_3^{1/2} \right) + \\ &+ \frac{\omega}{a_0} \tanh(\omega\pi) \left(\mu_7 V_{K3} Z_7^2(r) C_8^{(0)} - \mu_3 Z_3^2(r) C_4^{(0)} \right) + \\ &+ \frac{1}{2} l^2 g_s^{-1} f(\theta) \left(\prod_{i=3,7} e^{\xi a_0^{-1} - \xi a_0^{-1} Z_i^2} Z_i^{-9/8} \right) r^{-2} \left(\mu_7 V_{K3} Z_7^{1/2} - \mu_3 Z_3^{1/2} \right) v^2 . \qquad (4.33) \end{aligned}$$

The analysis goes through as above ,

$$\begin{aligned} \mu_7 \left[g_s^{-1} f(\theta) Z_3(r) Z_7(r) \left(Z_3^{-1/2} - \frac{V_*}{V_{K3}} Z_7^{-1/2} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] &\leq \mathcal{E} , \\ \mu_7 \left[\beta\, g_s^{-1} f(\theta) Z_3(r) Z_7(r) \left(Z_3^{-1/2} - \frac{V_*}{V_{K3}} Z_7^{-1/2} \right) - C_8 + C_4 \frac{V_*}{V_{K3}} \right] &\geq \mathcal{E} . \qquad (4.34) \end{aligned}$$

These inequalities provide a region of allowed energies for the probe to be at a given position. As was noted in section 3.2.2, in the $a_0 \to 0$ limit the probe analysis breaks down.

4.3 Exponential hierarchy

Finally let us compute the ratio between the D and $D-2$ dimensional Planck scales following the procedure outlined in our previous paper [1, 15]. In particular, the relation between the eight and ten dimensional scales is (using again $\rho = a_0^{-1}[1 + a_0 \log(r)]$)

$$M_8^6 = M_{10}^8 \int e^{6A+2B} \mathrm{d}\rho \mathrm{d}\theta = \pi l^2 M_{10}^8 \, e^{\xi/a_0} \rho_o^{9/16} \gamma \left(\frac{7}{16}, \rho_o \right), \qquad (4.35)$$

where $\rho_o(r_\mathrm{o})$ represents the upper limit of the *physically* allowed region $r \in [r_\mathrm{s}^+, r_\mathrm{o}]$; the integral vanishes in the lower limit, r_s^+. Clearly, the exponential hierarchy driven by ξ/a_0 remains essentially unchanged.

There is a formula analogous to (4.35) also in the case when our background is wrapped on a K3. However, depending on the energy of the probe, there are two possible cases; see figure 7. If $\mathcal{E}$ is larger than the maximum value of the lower limit function in the inequality (4.14), then $r_\mathrm{e} \leq r \leq r_\Phi$, and the analogue of eq. (4.35) has a single contribution. On the other hand, if $\mathcal{E}$ is less than that, then the probe is allowed in two disconnected regions: $r_\mathrm{e} \leq r \leq r_{\mathrm{o}1}$ and $r_{\mathrm{o}2} \leq r \leq r_\Phi$. All of this will however only modify the value of the coefficient in the analogue of eq. (4.35); the exponential hierarchy remains governed by e^{ξ/a_0}, as derived in ref. [15, 1].

5. Conclusions

In this paper we have presented a detailed analysis of the space-time properties of a certain family of non-supersymmetric vacua which can lead to an exponential hierarchy for the string coupling of $O(1)$. In particular we have identified a limit in which these non-supersymmetric vacua can be interpreted from the point of view of F-theory. In principle, the dynamical breaking of supersymmetry of these F-theory models of particle physics should be tied to the emergence of exponential hierarchy. It would be very interesting to understand this more precisely.

The exponential hierarchy is due to the presence of a classical naked singularity of our background. We have shown that this singularity can be resolved via a non-supersymmetric effect analogous to the enhançon mechanism, which in turn is based on a repulsive nature of the naked singularity as seen by quantum probes.

One potentially very exciting feature of our solution is that the constant ξ/a_0 which appears in the expression for the ratio of the D and $D-2$ dimensional Planck scales is undetermined and therefore is an input parameter. Thus, in principle one might wonder if we can have arbitrarily large exponential hierarchy. (This is similar to some recent discussion on the cosmological constant problem from a "holographic" point of view [49].) In our case, the emergence of exponential hierarchy might be tied to a dynamical supersymmetry breaking of a particular class of F-theory vacua. It is natural to expect that the value of ξ/a_0 is related to the supersymmetry breaking scale.

Finally, one aspect of classical gravitation analysis not addressed in this paper is the issue of classical stability. Recall that the general perturbation of the metric $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ satisfies (in a suitable gauge) the Lichnerowicz equation [50]

$$\Delta_L h_{\mu\nu} = [\delta_\mu^{\mu_1}\delta_\nu^{\nu_1}\Box + 2R_{\mu\ \ \nu}^{\ \mu_1\nu_1}]h_{\mu_1\nu_1} = 0\,. \tag{5.1}$$

The general perturbation is split into scalar, vector and tensor modes and the stability analysis is reduced to finding whether there exist any unstable modes for the perturbation. Work along these lines is in progress [51].

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